



1021_HyperSizer-Methods-Approach-FBD Tab Matrix Math.ppt

**Collier Research Corporation
Hampton, VA**

Objective



- ❑ 1st glimpse of HyperSizer Basic
- ❑ Get familiar with ABD matrix computations

Objective



- ❑ 1st glimpse of HyperSizer Basic
- ❑ Get familiar with ABD matrix computations
- ❑ Why?
 - ❑ HyperSizer smeared stiffness formulation



Outline



- ❑ HyperSizer panel stiffness approach

- ❑ Isotropic plate stiffness relations

- ❑ Metallic sheet examples
 - ❑ Mechanical loads
 - ❑ Thermal loads
 - ❑ Superimposed pressure





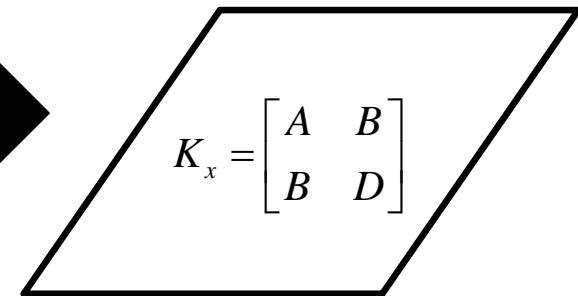
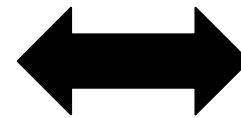
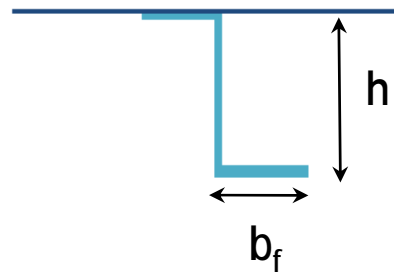
Panel Stiffness Approach

Panel Stiffness - Technical Approach



Local Stiffness

Global Stiffness



"smeared stiffness"

[45/90/90/-45/0/0/90/0]s

Panel Stiffness – Technical Approach



- Stiffened panel constitutive equation
 - [A] → membrane
 - [D] → bending
 - [B] → membrane-bending coupling

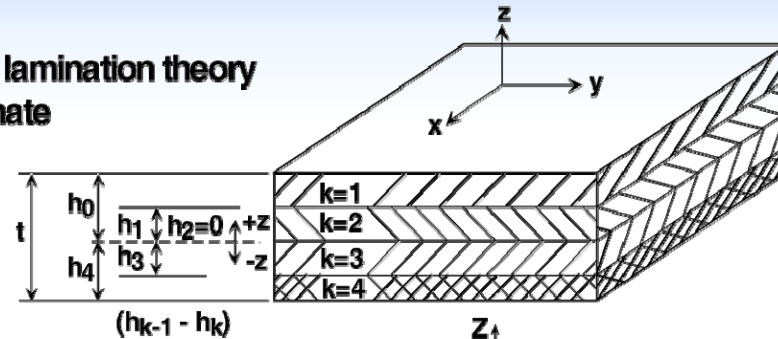
$$\begin{bmatrix} \vec{N} \\ \vec{M} \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B} & \mathbf{D} \end{bmatrix} \begin{bmatrix} \vec{\varepsilon} \\ \vec{\kappa} \end{bmatrix} - \begin{bmatrix} \vec{N}^T \\ \vec{M}^T \end{bmatrix}$$

Panel Stiffness - Technical Approach

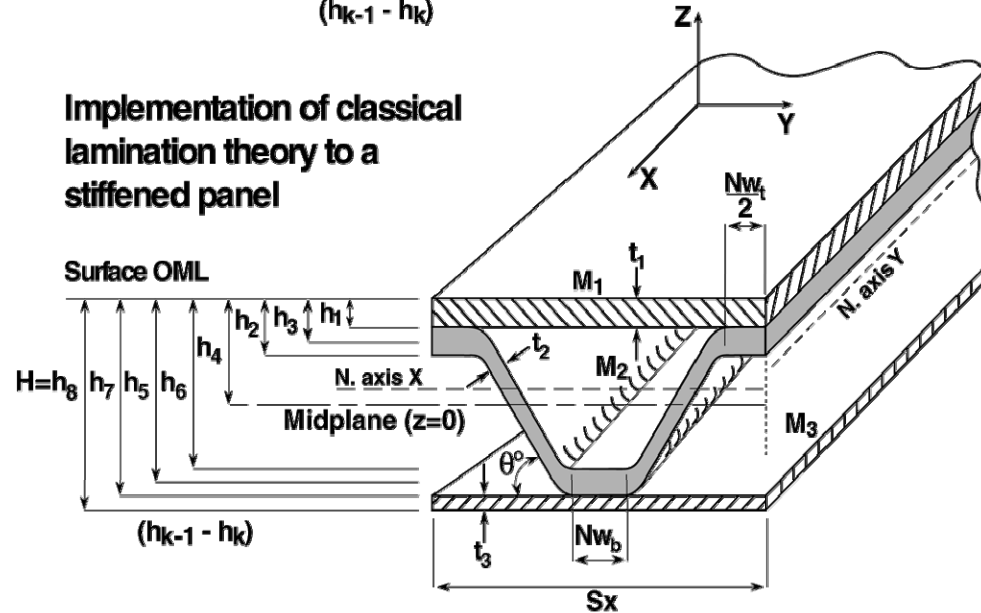


- Classical Lamination Theory extended to a represent any stiffened cross sectional shape

Classical lamination theory of a laminate



Implementation of classical lamination theory to a stiffened panel



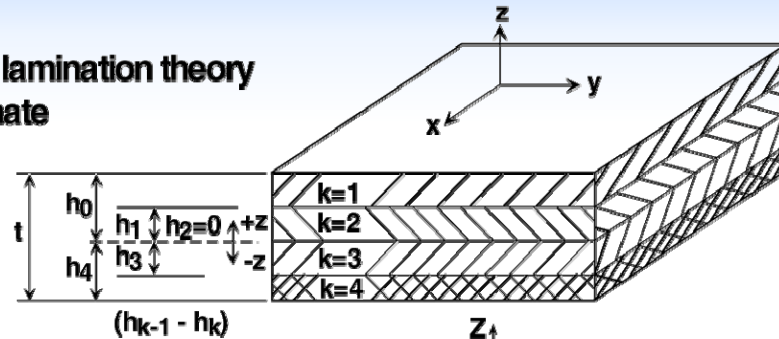
ILMNT FRMLTN(2)vertl Collier

Panel Stiffness - Technical Approach

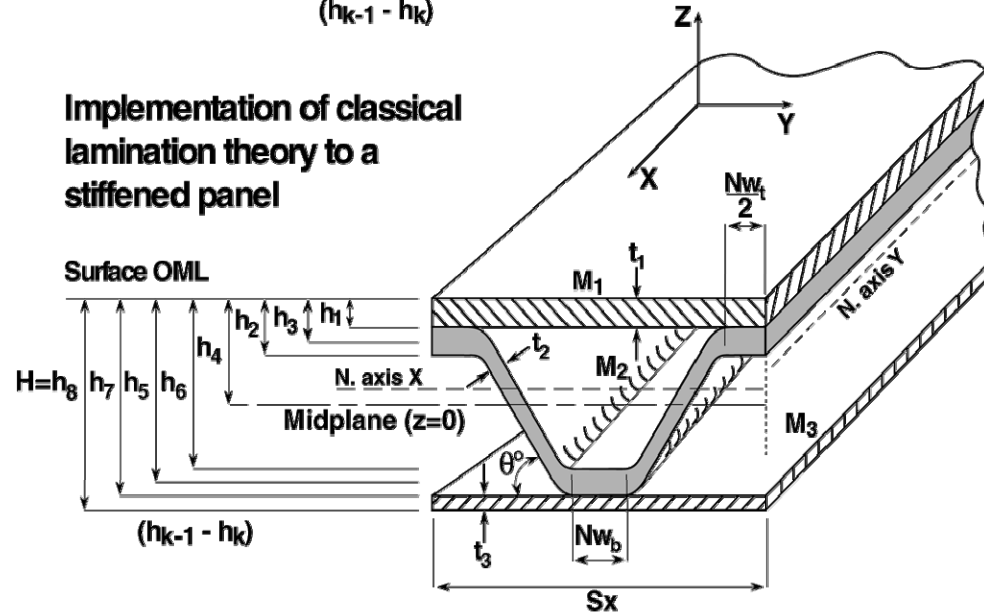


- Classical Lamination Theory extended to a represent any stiffened cross sectional shape
- General panel behaviors, are quantified with:
 - Stiffness terms [A], [B], [D]
 - Thermal coefficients [A^α], [B^α], [D^α]

Classical lamination theory of a laminate

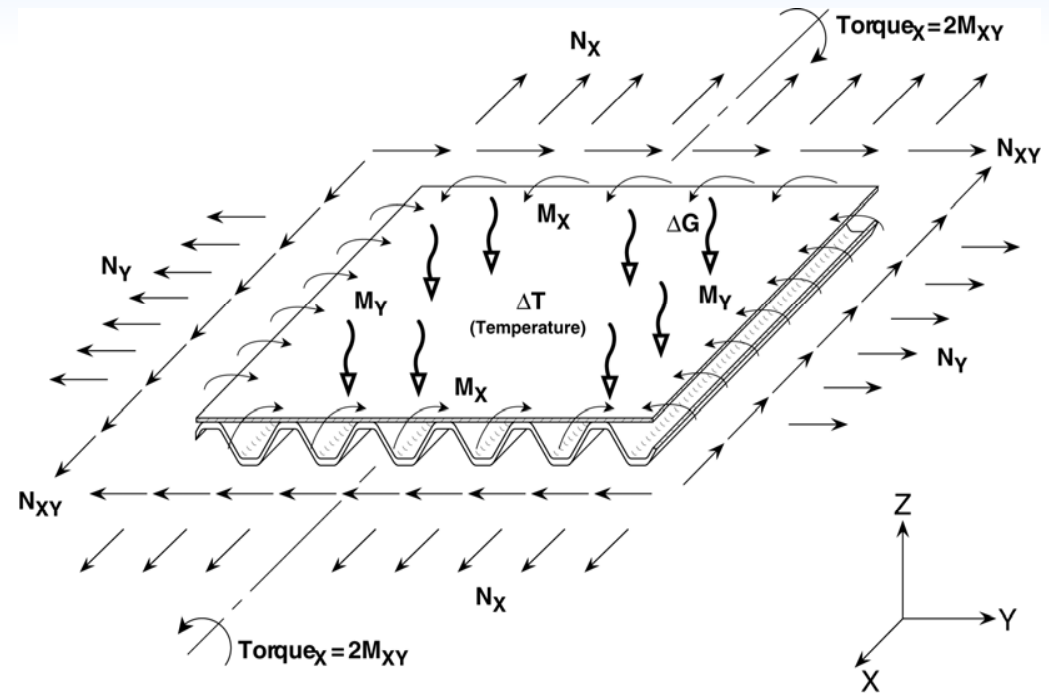


Implementation of classical lamination theory to a stiffened panel



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Free Body Diagram (FBD)



IST-SZICollier

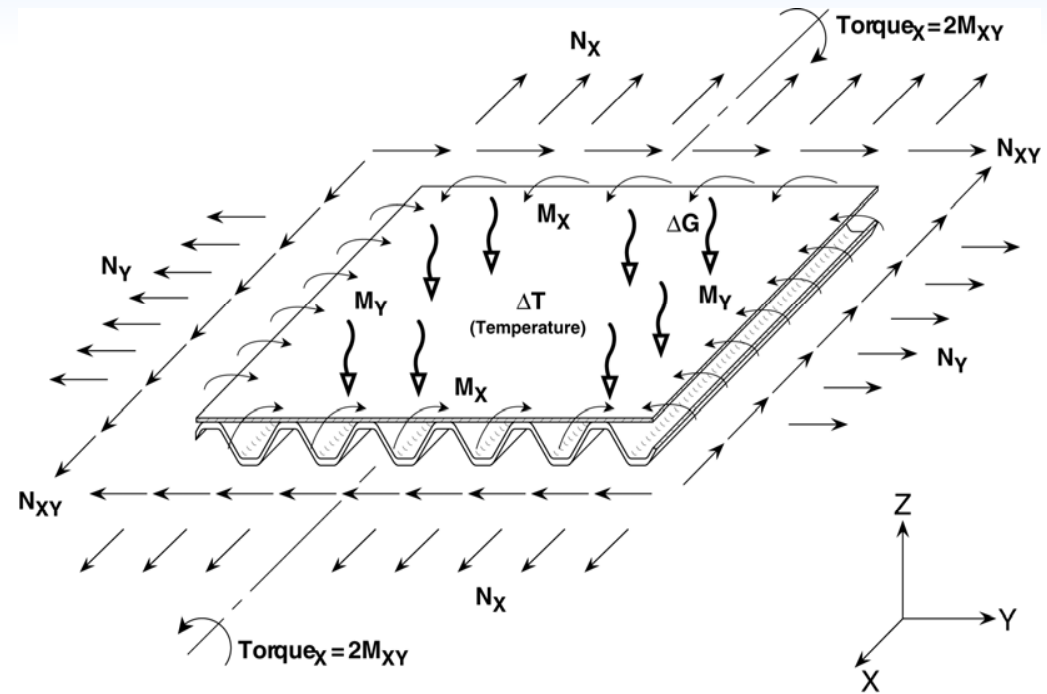
Thermoelastic Formulations



Free Body Diagram (FBD)



- **Balanced free-body loads**
 - FEA
 - User-defined input



IST-SZICollier

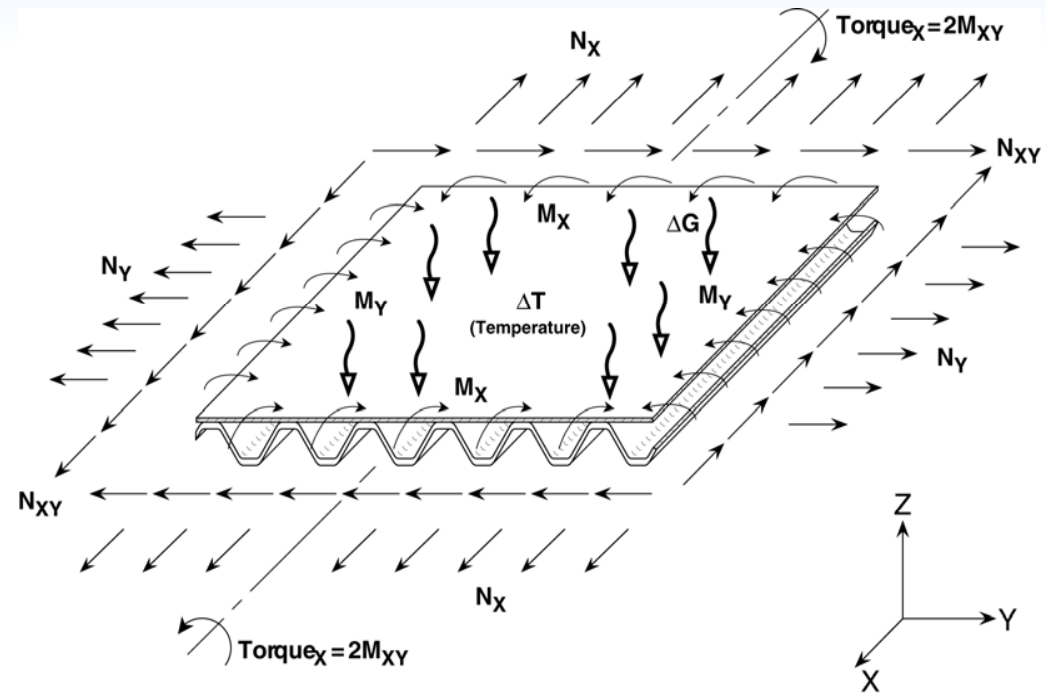
Thermoelastic Formulations



Free Body Diagram (FBD)



- **Balanced free-body loads**
 - FEA
 - User-defined input
- **Consistently applied thermoelastic formulations guarantee**
 - Equilibrium of forces



IST-SZICollier

Thermoelastic Formulations





Isotropic Plate Stiffness Relations

Isotropic Plate Stiffness



Plane Stress Constitutive Equations

□ Compliance

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} \frac{1}{E} & \frac{-\nu}{E} & 0 \\ \frac{-\nu}{E} & \frac{1}{E} & 0 \\ 0 & 0 & \frac{1}{G_{12}} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix}$$

Isotropic Plate Stiffness



Plane Stress Constitutive Equations

□ Compliance

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} \frac{1}{E} & \frac{-\nu}{E} & 0 \\ \frac{-\nu}{E} & \frac{1}{E} & 0 \\ 0 & 0 & \frac{1}{G_{12}} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix}$$



□ Stiffness

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} \frac{E}{1-\nu^2} & \frac{\nu E}{1-\nu^2} & 0 \\ \frac{\nu E}{1-\nu^2} & \frac{E}{1-\nu^2} & 0 \\ 0 & 0 & G_{12} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix}$$

Isotropic Plate Stiffness



Plane Stress Constitutive Equations

□ Compliance

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} \frac{1}{E} & \frac{-\nu}{E} & 0 \\ \frac{-\nu}{E} & \frac{1}{E} & 0 \\ 0 & 0 & \frac{1}{G_{12}} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix}$$



□ Stiffness

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} \frac{E}{1-\nu^2} & \frac{\nu E}{1-\nu^2} & 0 \\ \frac{\nu E}{1-\nu^2} & \frac{E}{1-\nu^2} & 0 \\ 0 & 0 & G_{12} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix}$$

Poisson term
for plates

$$\frac{1}{1-\nu^2}$$

ABD Matrix of Isotropic Plate



$$\begin{bmatrix} N_x \\ N_y \\ N_s \\ M_x \\ M_y \\ M_s \end{bmatrix} = \begin{bmatrix} \frac{Et}{1-\nu^2} & \frac{\nu Et}{1-\nu^2} & 0 & 0 & 0 & 0 \\ \frac{\nu Et}{1-\nu^2} & \frac{Et}{1-\nu^2} & 0 & 0 & 0 & 0 \\ 0 & 0 & Gt & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{Et^3}{12(1-\nu^2)} & \frac{\nu Et^3}{12(1-\nu^2)} & 0 \\ 0 & 0 & 0 & \frac{\nu Et^3}{12(1-\nu^2)} & \frac{Et^3}{12(1-\nu^2)} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{Gt^3}{12} \end{bmatrix} \begin{bmatrix} \epsilon_x^o \\ \epsilon_y^o \\ \gamma_{xy}^o \\ \kappa_x \\ \kappa_y \\ \kappa_z \end{bmatrix}$$

ABD Matrix of Isotropic Plate



$$\begin{bmatrix} N_x \\ N_y \\ N_s \\ M_x \\ M_y \\ M_s \end{bmatrix} = \begin{bmatrix} \frac{Et}{1-\nu^2} & \frac{\nu Et}{1-\nu^2} & 0 & 0 & 0 & 0 \\ \frac{\nu Et}{1-\nu^2} & \frac{Et}{1-\nu^2} & 0 & 0 & 0 & 0 \\ 0 & 0 & Gt & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{Et^3}{12(1-\nu^2)} & \frac{\nu Et^3}{12(1-\nu^2)} & 0 \\ 0 & 0 & 0 & \frac{\nu Et^3}{12(1-\nu^2)} & \frac{Et^3}{12(1-\nu^2)} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{Gt^3}{12} \end{bmatrix} \begin{bmatrix} \epsilon_x^o \\ \epsilon_y^o \\ \gamma_{xy}^o \\ \kappa_x \\ \kappa_y \\ \kappa_z \end{bmatrix}$$

ABD Matrix of Isotropic Plate



Membrane

$$A_{11} = A_{22} = \frac{Et}{1-\nu^2}$$

$$A_{12} = A_{21} = A_{11}\nu$$

$$A_{11}^{-1} = A_{22}^{-1} = \frac{1}{Et}$$

$$A_{12}^{-1} = A_{21}^{-1} = -A_{11}^{-1}\nu$$

Bending

$$D_{11} = D_{22} = \frac{Et^3}{12(1-\nu^2)}$$

$$D_{12} = D_{21} = D_{11}\nu$$

$$D_{11}^{-1} = D_{22}^{-1} = \frac{12}{Et^3}$$

$$D_{12}^{-1} = D_{21}^{-1} = -D_{11}^{-1}\nu$$



Examples

Isotropic Plate Force and Moment Exercises



Perform several exercises to verify the physics of the ABD matrix in using HyperSizer's operation user-defined loads

- Specified Strain
- Specified Force
- Uniform ΔT
- Through-Thickness ΔT
- Pressure



Set up for Demo Problem



Copy an Isotropic: 'AL 7075' and rename to 'FBD Example'

Set properties

$$E_c = E_t = 10 \text{ Msi}$$

$$G = 3.846154 \text{ Msi}$$

$$\nu = 0.3$$

$$\alpha = 12^{-6}$$

Setup Group sizing bounds thickness equal to .1"

Set Ultimate Load Factor = 1.0



Examples with Isotropic Plates



Isotropic Plate

$$E = 10 \text{ Msi}$$

$$\nu = 0.3$$

$$G = 3.846154 \text{ Msi}$$

$$\text{Plate Thickness, } t = 0.1''$$

$$\text{CTE, } \alpha = 12 \mu\text{in/in}$$

Membrane

$$A_{11} = \frac{Et}{1-\nu^2} = \frac{(10)(0.1)}{0.91}$$

$$= 1.0989 \times 10^6 \text{ lb/in}$$

Bending

$$D_{11} = \frac{Et^3}{12(1-\nu^2)} = \frac{(10)(0.1)^3}{12(0.91)}$$

$$= 915.8 \text{ lb/in}$$



The Free Body Diagram Tab



- Entry of Loads and Boundary Conditions

| Concepts | Design-to Loads | Failure | Buckling | Notes | | | | |
|---|--|--------------|---------------------|------------|------------|------------|------|------|
| Variables | FBD | Object Loads | Computed Properties | Options | | | | |
| Input (Per Load Case) | | | | | | | | |
| **ULTIMATE-MECHANICAL** Load Case #1 "one" (Mechanical Set #101, Thermal Set #201) | | | | | | | | |
| <input checked="" type="radio"/> Mechanical Load Set #101 "Load Set 101" <input type="radio"/> Thermal Load Set #201 "Load Set 201" | | Ref Temp | Temp | | | | | |
| <input type="radio"/> FEA Loads - Projects Only <input checked="" type="radio"/> User Loads Applied Unit Value For Strength Analysis For Buckling Analysis | | Pressure | TT Grad | | | | | |
| | Nx,ex | Ny,ey | Nxy,xyy | Mx,ex | My,ey | Mxy,xyy | Qx | Qy |
| | Load | Load | Load | Constraine | Constraine | Constraine | Load | Load |
| | Free | | | | | | | |
| | Constrained | | | | | | | |
| | Load | | | | | | | |
| | Deformation | | | | | | | |
| Superimposed Loads | | | | | | | | |
| <input type="checkbox"/> Panel Pressure | <input type="checkbox"/> Beam-Column Moments | | | | | | | |
| | | | | | | | | |
| Initial Imperfection <input type="text" value="0"/> | | | | | | | | |
| <input type="checkbox"/> Zero Out FEA Computed Moments | FIXED Boundary Condition | | | | | | | |
| MidSpan | Mx | My | Qx | Qy | | | | |
| EdgeCntr | 0 | 0 | 0 | 0 | | | | |
| | 0 | 0 | 0 | 0 | | | | |
| Point Free Body Diagram (Constant Forces) | | | | | | | | |
| | | | | | | | | |
| a (X length) <input type="text" value="30"/> | | | | | | | | |
| b (Y length) <input type="text" value="30"/> | | | | | | | | |
| Free Body Diagram Output (Controlling Factored Loadcase) | | | | | | | | |
| Controlling Analysis Load: BUCKLING | | | | | | | | |
| Virtual Loads | Nx,ex | Ny,ey | Nxy,xyy | Mx,ex | My,ey | Mxy,xyy | Qx | Qy |
| Design-to Loads | | | | | | | | |
| Design-to Deformation | 0 | 0 | 0 | 0 | 0 | 0 | | |



Ex 1 – Applied ϵ_x , Constrained ϵ_y



Input (Per Load Case)

LIMIT-MECHANICAL Load Case #1 "one" (Mechanical Set #101, Thermal Set #201)

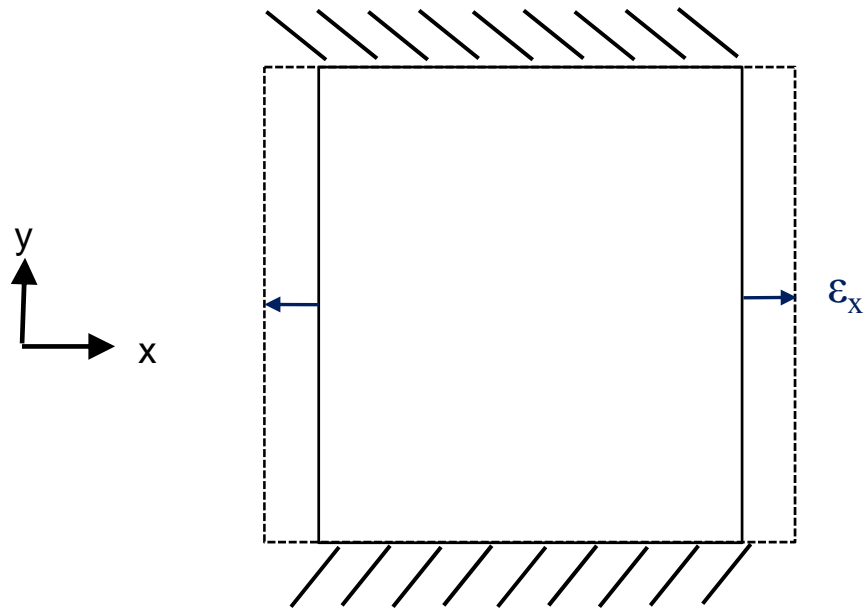
Mechanical Load Set #101 "Load Set 101"
 Thermal Load Set #201 "Load Set 201"

FEA Loads - Projects Only
 User Loads Applied Unit Value

For Strength Analysis
 For Buckling Analysis

| Nx, ϵ_x | Ny, ϵ_y | Nxy, γ_{xy} | Mx, κ_x | My, κ_y | Mxy, κ_{xy} | Qx | Qy |
|------------------|------------------|--------------------|----------------|----------------|--------------------|------|------|
| Deformati | Constrain | Constrain | Constrain | Constrain | Constrain | Load | Load |
| 0.001 | | | | | | | |
| 0.001 | | | | | | | |

Ref Temp Temp
 Pressure TT Grad



What will the loads look like?
Positive, negative or zero?

Ex 1 – Applied ϵ_x , Constrained ϵ_y



Input (Per Load Case)

LIMIT-MECHANICAL Load Case #1 "one" (Mechanical Set #101, Thermal Set #201)

Mechanical Load Set #101 "Load Set 101" Ref Temp Temp
 Thermal Load Set #201 "Load Set 201" Pressure TT Grad

FEA Loads - Projects Only

User Loads Applied Unit Value

| | Nx,εx | Ny,εy | Nxy,γxy | Mx,εx | My,εy | Mxy,εxy | Qx | Qy |
|-----------------------|-------|-------|---------|-------|-------|---------|----|----|
| For Strength Analysis | 0.001 | | | | | | | |
| For Buckling Analysis | 0.001 | | | | | | | |

$$N_x = A_{11}\epsilon_x + A_{12}\cancel{\epsilon_y}$$

$$= (1.0989 \times 10^6)(0.001)$$

$$= 1098.9$$

Set Ultimate Factor = 1.0

$$N_y = A_{21}\epsilon_x = \nu A_{11}\epsilon_x$$

$$= (0.3)(1.0989 \times 10^6)(0.001)$$

$$= 329.67$$

Free Body Diagram Output (Controlling Factored Loadcase)

Controlling Analysis Load: STRENGTH

| | Nx,εx | Ny,εy | Nxy,γxy | Mx,εx | My,εy | Mxy,εxy | Qx | Qy |
|-----------------------|--------|--------|---------|-------|-------|---------|----|----|
| Virtual Loads | 1098.9 | 329.67 | 0 | 0 | 0 | 0 | 0 | 0 |
| Design-to Loads | 1098.9 | 329.67 | 0 | 0 | 0 | 0 | 0 | 0 |
| Design-to Deformation | 0.001 | 0 | 0 | 0 | 0 | 0 | | |



Ex 2 – Applied N_x , Free N_y



Input (Per Load Case)

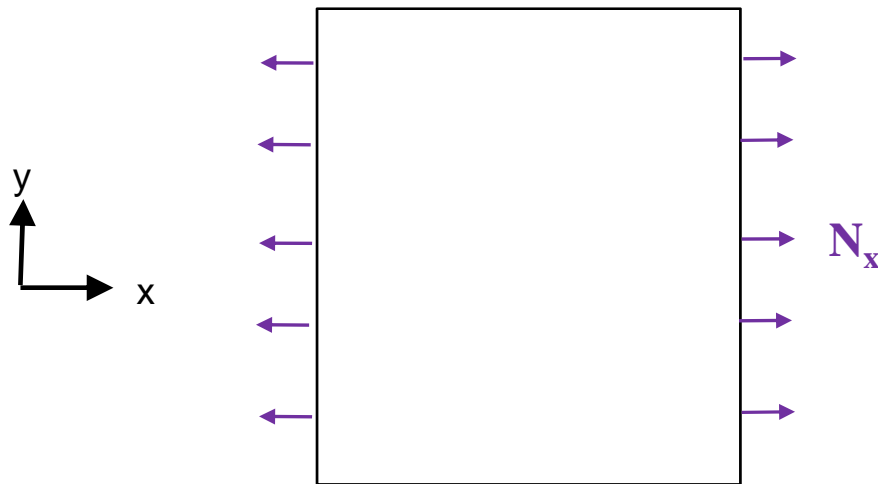
LIMIT-MECHANICAL Load Case #1 "one" (Mechanical Set #101, Thermal Set #201)

Mechanical Load Set #101 "Load Set 101" Ref Temp Temp
 Thermal Load Set #201 "Load Set 201" Pressure TT Grad

FEA Loads - Projects Only

User Loads Applied Unit Value For Strength Analysis For Buckling Analysis

| | N_x, ϵ_x | N_y, ϵ_y | N_{xy}, γ_{xy} | M_x, ϵ_x | M_y, ϵ_y | M_{xy}, ϵ_{xy} | Q_x | Q_y |
|-----------------------|-------------------|-------------------|-----------------------|-------------------|-------------------|-------------------------|-------|-------|
| | Load | Free | Constraine | Constraine | Constraine | Constraine | Load | Load |
| For Strength Analysis | 1000 | | | | | | | |
| For Buckling Analysis | 1000 | | | | | | | |



What will the strains look like?

Ex 2 – Applied N_x , Free N_y



Input (Per Load Case)

LIMIT-MECHANICAL Load Case #1 "one" (Mechanical Set #101, Thermal Set #201)

Mechanical Load Set #101 "Load Set 101"
 Thermal Load Set #201 "Load Set 201"

FEA Loads - Projects Only
 User Loads Applied Unit Value

For Strength Analysis
 For Buckling Analysis

| | N_x, ϵ_x | N_y, ϵ_y | N_{xy}, γ_{xy} | M_x, ϵ_x | M_y, ϵ_y | M_{xy}, ϵ_{xy} | Q_x | Q_y |
|-----------------------|-------------------|-------------------|-----------------------|-------------------|-------------------|-------------------------|--------|--------|
| Load | ▼ | Free ▼ | Constrained ▼ | Constrained ▼ | Constrained ▼ | Constrained ▼ | Load ▼ | Load ▼ |
| For Strength Analysis | 1000 | | | | | | | |
| For Buckling Analysis | 1000 | | | | | | | |

$$\epsilon_x = A_{11}^{-1} N_x + A_{12}^{-1} N_y \quad A_{11}^{-1} = \frac{1}{Et}$$

$$= \frac{1}{(10)(.1)} (1000) = 0.001$$

$$\epsilon_y = A_{21}^{-1} N_x + A_{22}^{-1} N_y \quad A_{21}^{-1} = \frac{-\nu}{Et}$$

$$= \frac{-(0.3)}{(10)(.1)} (1000) = -0.0003$$

Free Body Diagram Output (Controlling Factored Loadcase)

Controlling Analysis Load: STRENGTH

| | N_x, ϵ_x | N_y, ϵ_y | N_{xy}, γ_{xy} | M_x, ϵ_x | M_y, ϵ_y | M_{xy}, ϵ_{xy} | Q_x | Q_y |
|-----------------------|-------------------|-------------------|-----------------------|-------------------|-------------------|-------------------------|-------|-------|
| Virtual Loads | | | | | | | | |
| Design-to Loads | 1000 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Design-to Deformation | 0.001 | -2.999999E-04 | 0 | 0 | 0 | 0 | | |



Ex 3 – Applied κ_x , Constrained κ_y



Input (Per Load Case)

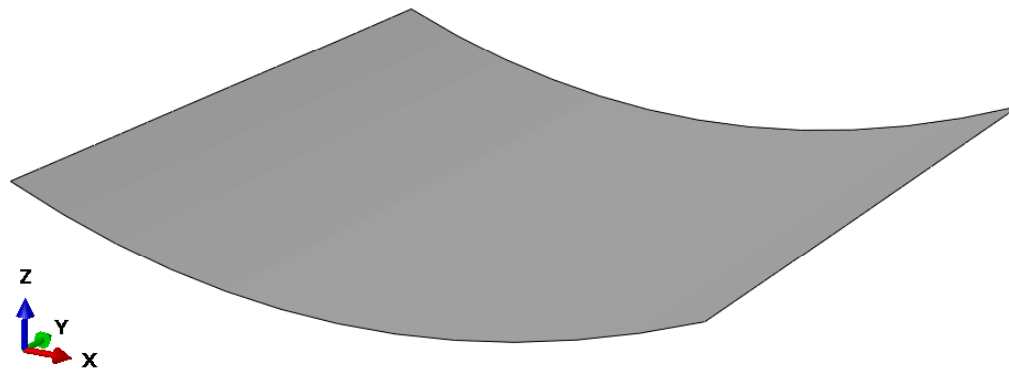
LIMIT-MECHANICAL Load Case #1 "one" (Mechanical Set #101, Thermal Set #201)

Mechanical Load Set #101 "Load Set 101" Ref Temp Temp
 Thermal Load Set #201 "Load Set 201" Pressure TT Grad

FEA Loads - Projects Only

User Loads Applied Unit Value For Strength Analysis For Buckling Analysis

| | Nx, ϵ_x | Ny, ϵ_y | Nxy, γ_{xy} | Mx, κ_x | My, κ_y | Mxy, ϵ_{xy} | Qx | Qy |
|--|------------------|------------------|--------------------|----------------|----------------|----------------------|------|------|
| | Free | Free | Constraine | Deformatic | Constraine | Constraine | Load | Load |
| | | | | 0.01 | | | | |
| | | | | 0.01 | | | | |



What will the loads look like?

Ex 3 – Applied κ_x , Constrained κ_y



Input (Per Load Case)

LIMIT-MECHANICAL Load Case #1 "one" (Mechanical Set #101, Thermal Set #201)

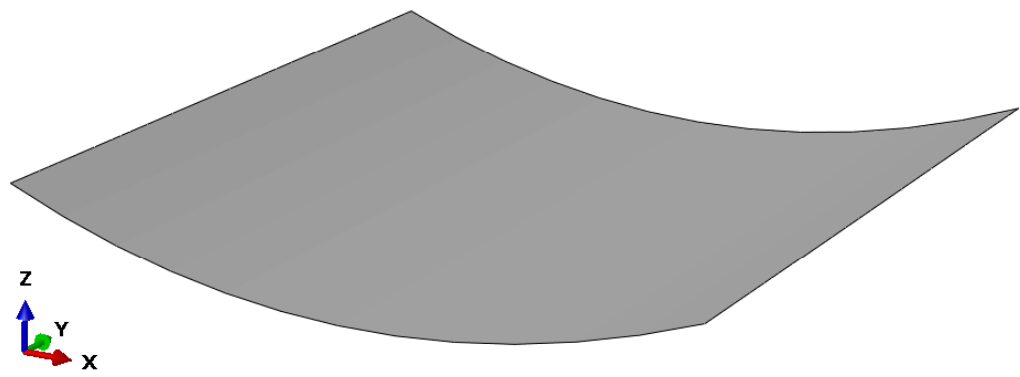
Mechanical Load Set #101 "Load Set 101"
 Thermal Load Set #201 "Load Set 201"

FEA Loads - Projects Only
 User Loads Applied Unit Value

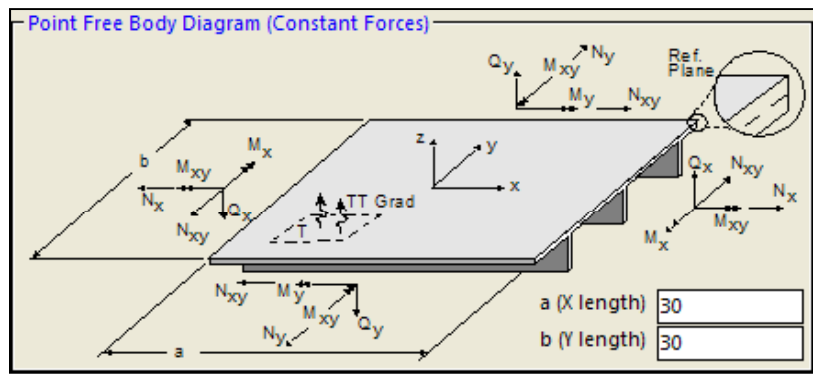
For Strength Analysis
 For Buckling Analysis

| | Nx,ex | Ny,ey | Nxy,xy | Mx,ix | My,iy | Mxy,ixy | Qx | Qy |
|--|-------|-------|------------|------------|------------|------------|------|------|
| | Free | Free | Constraine | Deformatic | Constraine | Constraine | Load | Load |
| | | | | 0.01 | | | | |
| | | | | 0.01 | | | | |

Ref Temp:
 Pressure: 0
 Temp:
 TT Grad:



What will the loads look like?



Ex 3 – Applied κ_x Constrained κ_y



Input (Per Load Case)

LIMIT-MECHANICAL Load Case #1 "one" (Mechanical Set #101, Thermal Set #201)

Mechanical Load Set #101 "Load Set 101" Ref Temp Temp

Thermal Load Set #201 "Load Set 201" Pressure TT Grad

FEA Loads - Projects Only

User Loads Applied Unit Value

| | Nx,εx | Ny,εy | Nxy,γxy | Mx,εx | My,εy | Mxy,εxy | Qx | Qy |
|-----------------------|-------|-------|------------|------------|------------|------------|------|------|
| For Strength Analysis | Free | Free | Constraine | Deformatic | Constraine | Constraine | Load | Load |
| For Buckling Analysis | | | | 0.01 | | | | |
| | | | | 0.01 | | | | |

$$Mx = D_{11}\kappa_x + D_{12}\cancel{\kappa_y}$$

$$= (915.8)(0.01)$$

$$= 9.158$$

$$My = D_{21}\kappa_x = \nu D_{11}\kappa_x$$

$$= (0.3)(915.8)(0.01)$$

$$= 2.747$$

Free Body Diagram Output (Controlling Factored Loadcase)

Controlling Analysis Load: STRENGTH

| | Nx,εx | Ny,εy | Nxy,γxy | Mx,εx | My,εy | Mxy,εxy | Qx | Qy |
|-----------------------|-------|-------|---------|---------|---------|---------|----|----|
| Virtual Loads | 0 | 0 | 0 | 9.15751 | 2.74725 | 0 | 0 | 0 |
| Design-to Loads | 0 | 0 | 0 | 9.15751 | 2.74725 | 0 | 0 | 0 |
| Design-to Deformation | 0 | 0 | 0 | 0.01 | 0 | 0 | | |



Ex 4 – Applied M_x , Free M_y



Input (Per Load Case)

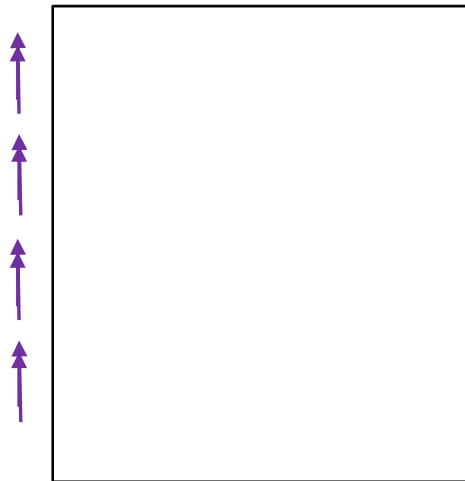
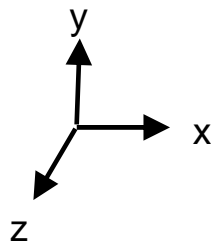
LIMIT-MECHANICAL Load Case #1 "one" (Mechanical Set #101, Thermal Set #201)

Mechanical Load Set #101 "Load Set 101" Ref Temp Temp
 Thermal Load Set #201 "Load Set 201" Pressure 0 TT Grad

FEA Loads - Projects Only

User Loads Applied Unit Value For Strength Analysis For Buckling Analysis

| | Nx, ϵ_x | Ny, ϵ_y | Nxy, γ_{xy} | Mx, ϵ_x | My, ϵ_y | Mxy, ϵ_{xy} | Qx | Qy |
|--|------------------|------------------|--------------------|------------------|------------------|----------------------|------|------|
| | Free | Free | Free | Load | Free | Constraine | Load | Load |
| | | | | 100 | | | | |
| | | | | 100 | | | | |



M_x

What will the strains look like?

Ex 4 – Applied M_x , Free M_y



Input (Per Load Case)

LIMIT-MECHANICAL Load Case #1 "one" (Mechanical Set #101, Thermal Set #201)

Mechanical Load Set #101 "Load Set 101" Ref Temp Temp
 Thermal Load Set #201 "Load Set 201" Pressure TT Grad

FEA Loads - Projects Only
 User Loads Applied Unit Value

| | Nx,ex | Ny,ey | Nxy,xy | Mx,xx | My,xy | Mxy,xy | Qx | Qy |
|-----------------------|-------|-------|--------|-------|-------|-------------|------|------|
| For Strength Analysis | Free | Free | Free | Load | Free | Constrained | Load | Load |
| For Buckling Analysis | | | | 100 | | | | |
| | | | | 100 | | | | |

$$\kappa_x = D_{11}^{-1}M_x + D_{12}^{-1}M_y \quad D_{11}^{-1} = \frac{12}{Et^3}$$

$$= \frac{12}{(10 \times 10^6)(.1)^3} (100) = 0.12$$

$$\kappa_y = D_{21}^{-1}M_x + D_{22}^{-1}M_y \quad D_{21}^{-1} = \frac{-12\nu}{Et^2}$$

$$= \frac{-12(0.3)}{(10 \times 10^6)(.1)^3} (100) = -0.036$$

Free Body Diagram Output (Controlling Factored Loadcase)

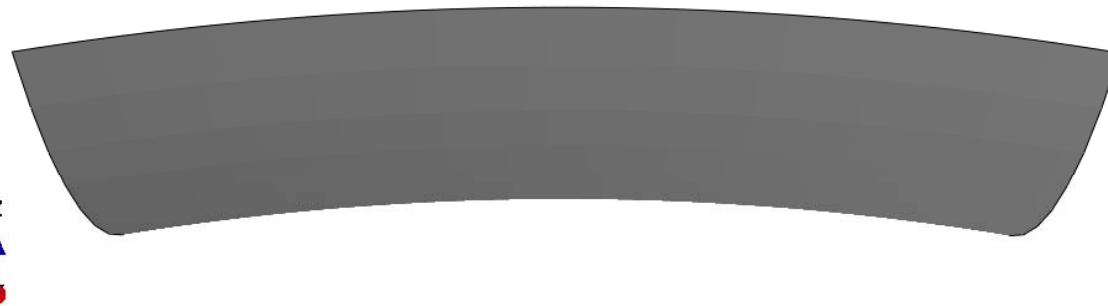
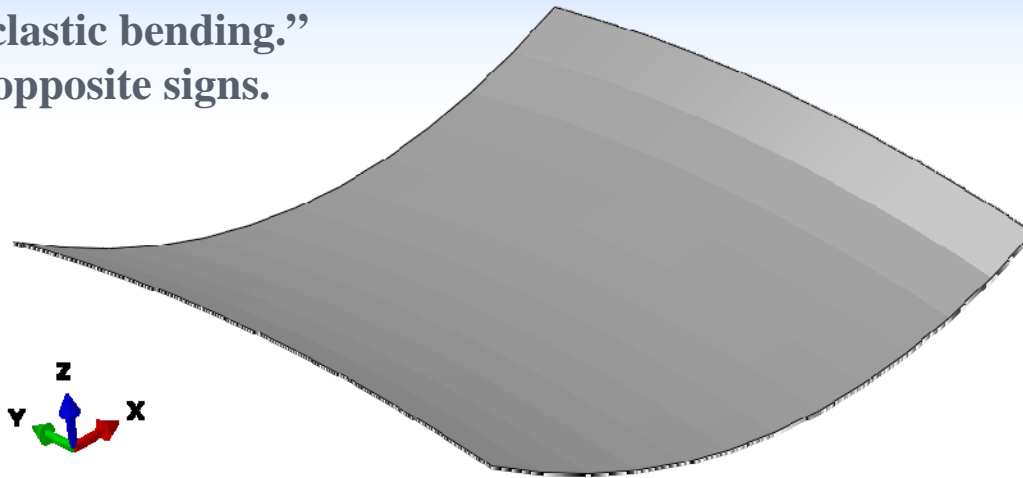
Controlling Analysis Load: STRENGTH

| | Nx,ex | Ny,ey | Nxy,xy | Mx,xx | My,xy | Mxy,xy | Qx | Qy |
|-----------------------|-------|-------|--------|-------|---------------|--------|----|----|
| Virtual Loads | | | | | | | | |
| Design-to Loads | 0 | 0 | 0 | 100 | 0 | 0 | 0 | 0 |
| Design-to Deformation | 0 | 0 | 0 | 0.12 | -3.599999E-02 | 0 | | |

Ex 4 – Applied M_x , Free M_y



Example of “anticlastic bending.”
Curvatures have opposite signs.



Free Body Diagram Output (Controlling Factored Loadcase)

Controlling Analysis Load: STRENGTH

| | N_x, ϵ_x | N_y, ϵ_y | N_{xy}, γ_{xy} | M_x, α_x | M_y, α_y | M_{xy}, α_{xy} | Q_x | Q_y |
|-----------------------|-------------------|-------------------|-----------------------|-----------------|-----------------|-----------------------|-------|-------|
| Virtual Loads | | | | | | | | |
| Design-to Loads | 0 | 0 | 0 | 100 | 0 | 0 | 0 | 0 |
| Design-to Deformation | 0 | 0 | 0 | 0.12 | -3.599999E-02 | 0 | | |

Ex 5 – Thermal: Applied ΔT Constrained



Input (Per Load Case)

ULTIMATE-THERMAL Load Case #1 "one" (Mechanical Set #101, Thermal Set #201)

Mechanical Load Set #101 "Load Set 101"

Thermal Load Set #201 "Load Set 201"

FEA Loads - Projects Only

User Loads Applied Unit Value

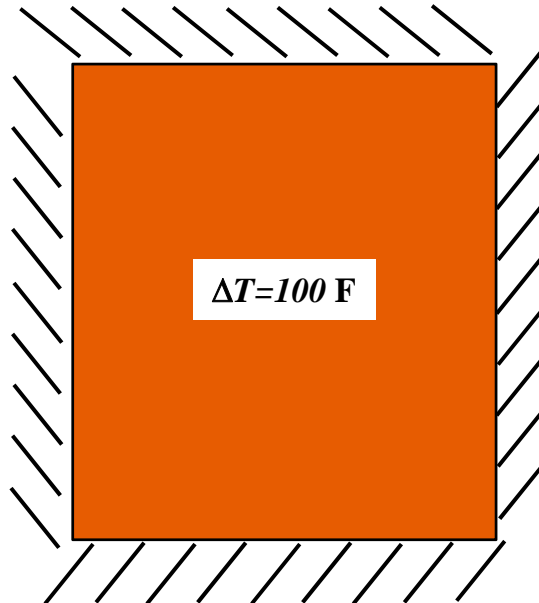
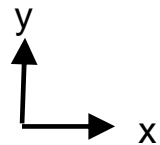
For Strength Analysis

For Buckling Analysis

| Nx, ex | Ny, ey | Nxy, γ_{xy} | Mx, ix | My, iy | Mxy, ixy | Qx | Qy |
|-------------|-------------|--------------------|-------------|-------------|-------------|------|------|
| Constrained | Constrained | Constrained | Constrained | Constrained | Constrained | Load | Load |
| | | | | | | | |

Ref Temp 100 Temp 200

Pressure TT Grad 0



What will the loads look like?
Positive, negative or zero?

Ex 5 –Thermal: Applied ΔT Constrained

Input (Per Load Case)

ULTIMATE-THERMAL Load Case #1 "one" (Mechanical Set #101, Thermal Set #201)

Mechanical Load Set #101 "Load Set 101"
 Thermal Load Set #201 "Load Set 201"
 FEA Loads - Projects Only
 User Loads Applied Unit Value

For Strength Analysis
 For Buckling Analysis

Ref Temp: 100 Temp: 200
 Pressure: TT Grad: 0

| | Nx, ϵ_x | Ny, ϵ_y | Nxy, γ_{xy} | Mx, ϵ_x | My, ϵ_y | Mxy, ϵ_{xy} | Qx | Qy |
|-------------|------------------|------------------|--------------------|------------------|------------------|----------------------|------|------|
| Constrained | Constrained | Constrained | Constrained | Constrained | Constrained | Constrained | Load | Load |
| | | | | | | | | |
| | | | | | | | | |

Free Body Diagram Output (Controlling Factored Loadcase)

Controlling Analysis Load: BUCKLING

| | Nx, ϵ_x | Ny, ϵ_y | Nxy, γ_{xy} | Mx, ϵ_x | My, ϵ_y | Mxy, ϵ_{xy} | Qx | Qy |
|-----------------------|------------------|------------------|--------------------|------------------|------------------|----------------------|----|----|
| Virtual Loads | -1714.29 | -1714.29 | 0 | 0 | 0 | 0 | 0 | 0 |
| Design-to Loads | -1714.29 | -1714.29 | 0 | 0 | 0 | 0 | 0 | 0 |
| Design-to Deformation | -0.0012 | -0.0012 | 0 | 0 | 0 | 0 | | |

Ex 5 –Thermal: Applied ΔT Constrained



Input (Per Load Case)

ULTIMATE-THERMAL Load Case #1 "one" (Mechanical Set #101, Thermal Set #201)

Mechanical Load Set #101 "Load Set 101"
 Thermal Load Set #201 "Load Set 201"
 FEA Loads - Projects Only
 User Loads Applied Unit Value

For Strength Analysis
 For Buckling Analysis

Ref Temp: 100 Temp: 200
 Pressure: TT Grad: 0

| | Nx, ϵ_x | Ny, ϵ_y | Nxy, γ_{xy} | Mx, ϵ_x | My, ϵ_y | Mxy, ϵ_{xy} | Qx | Qy |
|-----------------------|------------------|------------------|--------------------|------------------|------------------|----------------------|------|------|
| Applied Unit Value | Constrained | Constrained | Constrained | Constrained | Constrained | Constrained | Load | Load |
| For Strength Analysis | | | | | | | | |
| For Buckling Analysis | | | | | | | | |

Thermal Strain

$$\begin{aligned} \epsilon_x^T &= \epsilon_y^T = \alpha \Delta T \\ &= (12 \times 10^{-6})(100) \\ &= 0.0012 \end{aligned}$$

Free Body Diagram Output (Controlling Factored Loadcase)

Controlling Analysis Load: BUCKLING

| | Nx, ϵ_x | Ny, ϵ_y | Nxy, γ_{xy} | Mx, ϵ_x | My, ϵ_y | Mxy, ϵ_{xy} | Qx | Qy |
|-----------------------|------------------|------------------|--------------------|------------------|------------------|----------------------|----|----|
| Virtual Loads | -1714.29 | -1714.29 | 0 | 0 | 0 | 0 | 0 | 0 |
| Design-to Loads | -1714.29 | -1714.29 | 0 | 0 | 0 | 0 | 0 | 0 |
| Design-to Deformation | -0.0012 | -0.0012 | 0 | 0 | 0 | 0 | | |



Ex 5 –Thermal: Applied ΔT Constrained



Input (Per Load Case)

ULTIMATE-THERMAL Load Case #1 "one" (Mechanical Set #101, Thermal Set #201)

Mechanical Load Set #101 "Load Set 101"
 Thermal Load Set #201 "Load Set 201"

FEA Loads - Projects Only
 User Loads Applied Unit Value

For Strength Analysis
 For Buckling Analysis

| | | | | | | | |
|------------------|------------------|--------------------|------------------|------------------|----------------------|------|------|
| Nx, ϵ_x | Ny, ϵ_y | Nxy, γ_{xy} | Mx, ϵ_x | My, ϵ_y | Mxy, ϵ_{xy} | Qx | Qy |
| Constrained | Constrained | Constrained | Constrained | Constrained | Constrained | Load | Load |
| | | | | | | | |

Ref Temp: 100 Temp: 200
 Pressure: TT Grad: 0

Thermal Strain

$$\begin{aligned} \epsilon_x^T &= \epsilon_y^T = \alpha \Delta T \\ &= (12 \times 10^{-6})(100) \\ &= 0.0012 \end{aligned}$$

Strain Actual (Computed Properties Tab)

| Deformation | |
|-------------|---|
| Strain X | 0 |
| Strain Y | 0 |

Design-To Strain

$$\begin{aligned} \epsilon_x^{Design-To} &= \cancel{\epsilon_x^{Actual}} - \epsilon_x^T \\ &= -0.0012 \end{aligned}$$

Free Body Diagram Output (Controlling Factored Loadcase)

Controlling Analysis Load: BUCKLING

| | Nx, ϵ_x | Ny, ϵ_y | Nxy, γ_{xy} | Mx, ϵ_x | My, ϵ_y | Mxy, ϵ_{xy} | Qx | Qy |
|-----------------------|------------------|------------------|--------------------|------------------|------------------|----------------------|----|----|
| Virtual Loads | -1714.29 | -1714.29 | 0 | 0 | 0 | 0 | 0 | 0 |
| Design-to Loads | -1714.29 | -1714.29 | 0 | 0 | 0 | 0 | 0 | 0 |
| Design-to Deformation | -0.0012 | -0.0012 | 0 | 0 | 0 | 0 | | |



Ex 5 –Thermal: Applied ΔT Constrained



Input (Per Load Case)

ULTIMATE-THERMAL Load Case #1 "one" (Mechanical Set #101, Thermal Set #201)

Mechanical Load Set #101 "Load Set 101"
 Thermal Load Set #201 "Load Set 201"
 FEA Loads - Projects Only
 User Loads Applied Unit Value

For Strength Analysis
 For Buckling Analysis

Ref Temp: 100 Temp: 200
 Pressure: TT Grad: 0

| | Nx, ϵ_x | Ny, ϵ_y | Nxy, γ_{xy} | Mx, ϵ_x | My, ϵ_y | Mxy, ϵ_{xy} | Qx | Qy |
|-------------|------------------|------------------|--------------------|------------------|------------------|----------------------|------|------|
| Constrained | Constrained | Constrained | Constrained | Constrained | Constrained | Constrained | Load | Load |
| | | | | | | | | |

Thermal Strain

$$\begin{aligned} \epsilon_x^T &= \epsilon_y^T = \alpha \Delta T \\ &= (12 \times 10^{-6})(100) \\ &= 0.0012 \end{aligned}$$

Strain Actual (Computed Properties Tab)

| Deformation | |
|-------------|---|
| Strain X | 0 |
| Strain Y | 0 |

Design-To Strain

$$\begin{aligned} \epsilon_x^{Design-To} &= \cancel{\epsilon_x^{Actual}} - \epsilon_x^T \\ &= -0.0012 \end{aligned}$$

Design-To Force

$$\begin{aligned} N_x^{Design-To} &= A_{11} \epsilon_x^{Design-To} + A_{12} \epsilon_y^{Design-To} \\ &= (1.0989 \times 10^6)(-0.0012) + (0.3)(1.0989 \times 10^6)(-0.0012) \\ &= -1714.3 \end{aligned}$$

Free Body Diagram Output (Controlling Factored Loadcase)

Controlling Analysis Load: BUCKLING

| | Nx, ϵ_x | Ny, ϵ_y | Nxy, γ_{xy} | Mx, ϵ_x | My, ϵ_y | Mxy, ϵ_{xy} | Qx | Qy |
|-----------------------|------------------|------------------|--------------------|------------------|------------------|----------------------|----|----|
| Virtual Loads | -1714.29 | -1714.29 | 0 | 0 | 0 | 0 | 0 | 0 |
| Design-to Loads | -1714.29 | -1714.29 | 0 | 0 | 0 | 0 | 0 | 0 |
| Design-to Deformation | -0.0012 | -0.0012 | 0 | 0 | 0 | 0 | | |



Ex 6 – Thermal: Applied ΔT , Edges Free



Input (Per Load Case)

ULTIMATE-THERMAL Load Case #1 "one" (Mechanical Set #101, Thermal Set #201)

Mechanical Load Set #101 "Load Set 101"

Thermal Load Set #201 "Load Set 201"

Ref Temp 100 Temp 200

Pressure TT Grad 0

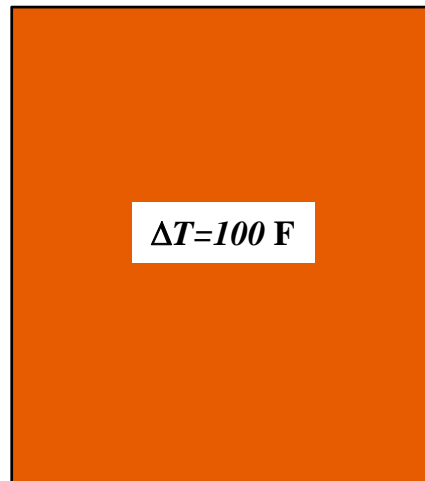
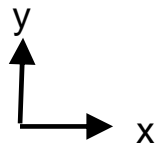
FEA Loads - Projects Only

User Loads Applied Unit Value

| Nx, sx | Ny, sy | Nbz, syz | Mx, sx | My, sy | Mbz, syz | Qx | Qy |
|--------|--------|----------|--------|--------|----------|------|------|
| Free | Free | Free | Free | Free | Free | Load | Load |
| | | | | | | | |
| | | | | | | | |

For Strength Analysis

For Buckling Analysis



What will the loads look like?
Positive, negative or zero?

Ex 6 –Thermal: Applied ΔT Free



Input (Per Load Case)

ULTIMATE-THERMAL Load Case #1 "one" (Mechanical Set #101, Thermal Set #201)

Mechanical Load Set #101 "Load Set 101"
 Thermal Load Set #201 "Load Set 201"

Ref Temp: 100 Temp: 200
 Pressure: TT Grad: 0

FEA Loads - Projects Only
 User Loads Applied Unit Value

| | Nx, ϵ_x | Ny, ϵ_y | Nxy, γ_{xy} | Mx, κ_x | My, κ_y | Mxy, κ_{xy} | Qx | Qy |
|-----------------------|------------------|------------------|--------------------|----------------|----------------|--------------------|------|------|
| For Strength Analysis | Free | Free | Free | Free | Free | Free | Load | Load |
| For Buckling Analysis | | | | | | | | |

Free Body Diagram Output (Controlling Factored Loadcase)

Controlling Analysis Load: BUCKLING

| | Nx, ϵ_x | Ny, ϵ_y | Nxy, γ_{xy} | Mx, κ_x | My, κ_y | Mxy, κ_{xy} | Qx | Qy |
|-----------------------|------------------|------------------|--------------------|----------------|----------------|--------------------|----|----|
| Virtual Loads | | | | | | | | |
| Design-to Loads | | | | | | | | |
| Design-to Deformation | 0 | 0 | 0 | 0 | 0 | 0 | | |



Ex 6 –Thermal: Applied ΔT Free



Input (Per Load Case)

ULTIMATE-THERMAL Load Case #1 "one" (Mechanical Set #101, Thermal Set #201)

Mechanical Load Set #101 "Load Set 101"
 Thermal Load Set #201 "Load Set 201"

FEA Loads - Projects Only
 User Loads Applied Unit Value

For Strength Analysis
 For Buckling Analysis

| | | | |
|--------------------|------|----------------------|------|
| Ref Temp | 100 | Temp | 200 |
| Pressure | | TT Grad | 0 |
| Nx, ϵ_x | Free | Ny, ϵ_y | Free |
| Nxy, γ_{xy} | Free | Mx, ϵ_x | Free |
| My, ϵ_y | Free | Mxy, ϵ_{xy} | Free |
| Qx | Load | Qy | Load |

Thermal Strain

$$\begin{aligned} \epsilon_x^T &= \epsilon_y^T = \alpha \Delta T \\ &= (12 \times 10^{-6})(100) \\ &= 0.0012 \end{aligned}$$

Free Body Diagram Output (Controlling Factored Loadcase)

Controlling Analysis Load: BUCKLING

| | Nx, ϵ_x | Ny, ϵ_y | Nxy, γ_{xy} | Mx, ϵ_x | My, ϵ_y | Mxy, ϵ_{xy} | Qx | Qy |
|-----------------------|------------------|------------------|--------------------|------------------|------------------|----------------------|----|----|
| Virtual Loads | | | | | | | | |
| Design-to Loads | | | | | | | | |
| Design-to Deformation | 0 | 0 | 0 | 0 | 0 | 0 | | |



Ex 6 –Thermal: Applied ΔT Free



Input (Per Load Case)

ULTIMATE-THERMAL Load Case #1 "one" (Mechanical Set #101, Thermal Set #201)

Mechanical Load Set #101 "Load Set 101"
 Thermal Load Set #201 "Load Set 201"

Ref Temp Temp
 Pressure TT Grad

| | Nx,ax | Ny,ay | Nxy,xy | Mx,ax | My,ay | Mxy,axy | Qx | Qy |
|--|-------|-------|--------|-------|-------|---------|------|------|
| <input checked="" type="radio"/> User Loads Applied Unit Value | Free | Free | Free | Free | Free | Free | Load | Load |
| For Strength Analysis | | | | | | | | |
| For Buckling Analysis | | | | | | | | |

Thermal Strain

$$\begin{aligned} \epsilon_x^T &= \epsilon_y^T = \alpha \Delta T \\ &= (12 \times 10^{-6})(100) \\ &= 0.0012 \end{aligned}$$

Strain Actual (Computed Properties Tab)

| Deformation | |
|-------------|-------------------------------------|
| Strain X | <input type="text" value="0.0012"/> |
| Strain Y | <input type="text" value="0.0012"/> |

Free Body Diagram Output (Controlling Factored Loadcase)

Controlling Analysis Load: BUCKLING

| | Nx,ax | Ny,ay | Nxy,xy | Mx,ax | My,ay | Mxy,axy | Qx | Qy |
|-----------------------|-------|-------|--------|-------|-------|---------|----|----|
| Virtual Loads | | | | | | | | |
| Design-to Loads | | | | | | | | |
| Design-to Deformation | 0 | 0 | 0 | 0 | 0 | 0 | | |



Ex 6 –Thermal: Applied ΔT Free



Input (Per Load Case)

ULTIMATE-THERMAL Load Case #1 "one" (Mechanical Set #101, Thermal Set #201)

Mechanical Load Set #101 "Load Set 101"
 Thermal Load Set #201 "Load Set 201"

FEA Loads - Projects Only
 User Loads Applied Unit Value

For Strength Analysis
 For Buckling Analysis

| | | | |
|--------------------|------|----------------------|------|
| Ref Temp | 100 | Temp | 200 |
| Pressure | | TT Grad | 0 |
| Nx, ϵ_x | Free | Ny, ϵ_y | Free |
| Nxy, γ_{xy} | Free | Mx, ϵ_x | Free |
| My, ϵ_y | Free | Mxy, ϵ_{xy} | Free |
| Qx | Load | Qy | Load |

Thermal Strain

$$\begin{aligned} \epsilon_x^T &= \epsilon_y^T = \alpha \Delta T \\ &= (12 \times 10^{-6})(100) \\ &= 0.0012 \end{aligned}$$

Strain Actual (Computed Properties Tab)

| Deformation | |
|-------------|--------|
| Strain X | 0.0012 |
| Strain Y | 0.0012 |

Design-To Strain

$$\begin{aligned} \epsilon_x^{Actual} &= \epsilon_x^T \\ \epsilon_x^{Design-To} &= \epsilon_x^{Actual} - \epsilon_x^T \\ &= 0.0 \end{aligned}$$

Free Body Diagram Output (Controlling Factored Loadcase)

Controlling Analysis Load: BUCKLING

| | Nx, ϵ_x | Ny, ϵ_y | Nxy, γ_{xy} | Mx, ϵ_x | My, ϵ_y | Mxy, ϵ_{xy} | Qx | Qy |
|-----------------------|------------------|------------------|--------------------|------------------|------------------|----------------------|----|----|
| Virtual Loads | | | | | | | | |
| Design-to Loads | | | | | | | | |
| Design-to Deformation | 0 | 0 | 0 | 0 | 0 | 0 | | |



Ex 6 –Thermal: Applied ΔT Free



Input (Per Load Case)

ULTIMATE-THERMAL Load Case #1 "one" (Mechanical Set #101, Thermal Set #201)

Mechanical Load Set #101 "Load Set 101"
 Thermal Load Set #201 "Load Set 201"
 FEA Loads - Projects Only
 User Loads Applied Unit Value

Ref Temp: 100 Temp: 200
 Pressure: TT Grad: 0

| | Nx,xx | Ny,yy | Nxy,xy | Mx,xx | My,yy | Mxy,xy | Qx | Qy |
|-----------------------|-------|-------|--------|-------|-------|--------|------|------|
| For Strength Analysis | Free | Free | Free | Free | Free | Free | Load | Load |
| For Buckling Analysis | | | | | | | | |

Thermal Strain

$$\begin{aligned} \epsilon_x^T &= \epsilon_y^T = \alpha \Delta T \\ &= (12 \times 10^{-6})(100) \\ &= 0.0012 \end{aligned}$$

Strain Actual (Computed Properties Tab)

| Deformation | |
|-------------|--------|
| Strain X | 0.0012 |
| Strain Y | 0.0012 |

Design-To Strain

$$\begin{aligned} \epsilon_x^{Actual} &= \epsilon_x^T \\ \epsilon_x^{Design-To} &= \epsilon_x^{Actual} - \epsilon_x^T \\ &= 0.0 \end{aligned}$$

Design-To Force

$$\begin{aligned} N_x^{Design-To} &= A_{11} \cancel{\epsilon_x^{Design-To}} + A_{12} \cancel{\epsilon_y^{Design-To}} \\ &= 0.0 \end{aligned}$$

Free Body Diagram Output (Controlling Factored Loadcase)

Controlling Analysis Load: BUCKLING

| | Nx,xx | Ny,yy | Nxy,xy | Mx,xx | My,yy | Mxy,xy | Qx | Qy |
|-----------------------|-------|-------|--------|-------|-------|--------|----|----|
| Virtual Loads | | | | | | | | |
| Design-to Loads | | | | | | | | |
| Design-to Deformation | 0 | 0 | 0 | 0 | 0 | 0 | | |



Ex 7 – Applied ΔT , Partially Constrained



Input (Per Load Case)

ULTIMATE-THERMAL Load Case #1 "one" (Mechanical Set #101, Thermal Set #201)

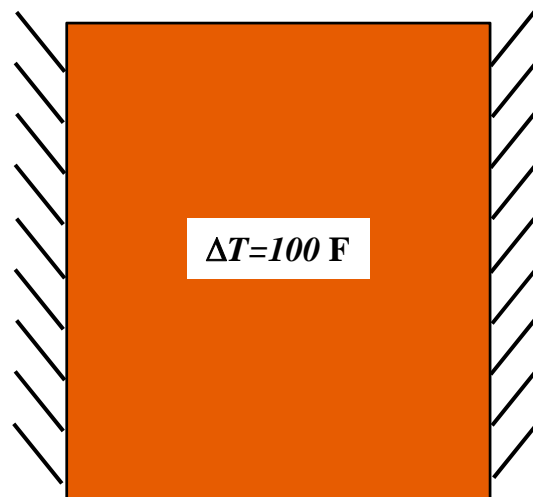
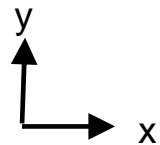
Mechanical Load Set #101 "Load Set 101" Ref Temp 100 Temp 200

Thermal Load Set #201 "Load Set 201" Pressure TT Grad 0

FEA Loads - Projects Only

User Loads Applied Unit Value

| | Nx, ex | Ny, ey | Nbz, ezy | Mx, ex | My, ey | Mbz, ezy | Qx | Qy |
|-----------------------|-------------|--------|----------|--------|--------|----------|------|------|
| For Strength Analysis | Constrained | Free | Free | Free | Free | Free | Load | Load |
| For Buckling Analysis | | | | | | | | |



What will the loads look like?
Positive, negative or zero?

Ex 7 -Applied ΔT Partially Constrained



Input (Per Load Case)

ULTIMATE-THERMAL Load Case #1 "one" (Mechanical Set #101, Thermal Set #201)

Mechanical Load Set #101 "Load Set 101"
 Thermal Load Set #201 "Load Set 201"

FEA Loads - Projects Only
 User Loads

Applied Unit Value

For Strength Analysis

For Buckling Analysis

| | | | |
|----------------|-------------|-----------------|------|
| Ref Temp | 100 | Temp | 200 |
| Pressure | | TT Grad | 0 |
| Nx, <i>ex</i> | Constrained | Ny, <i>ey</i> | Free |
| Nxy, <i>xy</i> | Free | Mx, <i>ix</i> | Free |
| My, <i>iy</i> | Free | Mxy, <i>ixy</i> | Free |
| Qx | Load | Qy | Load |

Free Body Diagram Output (Controlling Factored Loadcase)

Controlling Analysis Load: BUCKLING

| | Nx, <i>ex</i> | Ny, <i>ey</i> | Nxy, <i>xy</i> | Mx, <i>ix</i> | My, <i>iy</i> | Mxy, <i>ixy</i> | Qx | Qy |
|-----------------------|---------------|---------------|----------------|---------------|---------------|-----------------|----|----|
| Virtual Loads | -1.200 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Design-to Loads | -1.200 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Design-to Deformation | -0.0012 | 3.599999E-04 | 0 | 0 | 0 | 0 | | |



Ex 7 –Applied ΔT Partially Constrained



Input (Per Load Case)

ULTIMATE-THERMAL Load Case #1 "one" (Mechanical Set #101, Thermal Set #201)

Mechanical Load Set #101 "Load Set 101"
 Thermal Load Set #201 "Load Set 201"

FEA Loads - Projects Only
 User Loads Applied Unit Value

For Strength Analysis
 For Buckling Analysis

Ref Temp: 100 Temp: 200
 Pressure: TT Grad: 0

| | Nx, ϵ_x | Ny, ϵ_y | Nxy, γ_{xy} | Mx, ϵ_x | My, ϵ_y | Mxy, ϵ_{xy} | Qx | Qy |
|-----------------------|------------------|------------------|--------------------|------------------|------------------|----------------------|------|------|
| Applied Unit Value | Constrained | Free | Free | Free | Free | Free | Load | Load |
| For Strength Analysis | | | | | | | | |
| For Buckling Analysis | | | | | | | | |

Thermal Strain

$$\begin{aligned} \epsilon_x^T &= \epsilon_y^T = \alpha \Delta T \\ &= (12 \times 10^{-6})(100) \\ &= 0.0012 \end{aligned}$$

Free Body Diagram Output (Controlling Factored Loadcase)

Controlling Analysis Load: BUCKLING

| | Nx, ϵ_x | Ny, ϵ_y | Nxy, γ_{xy} | Mx, ϵ_x | My, ϵ_y | Mxy, ϵ_{xy} | Qx | Qy |
|-----------------------|------------------|------------------|--------------------|------------------|------------------|----------------------|----|----|
| Virtual Loads | -1200 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Design-to Loads | -1200 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Design-to Deformation | -0.0012 | 3.599999E-04 | 0 | 0 | 0 | 0 | | |

Ex 7 –Applied ΔT Partially Constrained



Input (Per Load Case)

ULTIMATE-THERMAL Load Case #1 "one" (Mechanical Set #101, Thermal Set #201)

Mechanical Load Set #101 "Load Set 101"
 Thermal Load Set #201 "Load Set 201"

FEA Loads - Projects Only
 User Loads Applied Unit Value

For Strength Analysis
 For Buckling Analysis

| | | | |
|----------|-------------|----------|------|
| Ref Temp | 100 | Temp | 200 |
| Pressure | | TT Grad | 0 |
| Nx, ex | Constrained | Ny, ey | Free |
| Nxy, yxy | Free | Mx, ix | Free |
| My, iy | Free | Mxy, ixy | Free |
| Qx | Load | Qy | Load |

Thermal Strain

$$\begin{aligned} \epsilon_x^T &= \epsilon_y^T = \alpha \Delta T \\ &= (12 \times 10^{-6})(100) \\ &= 0.0012 \end{aligned}$$

Strain Actual (Computed Properties Tab)

| Deformation | |
|-------------|---------|
| Strain X | 0 |
| Strain Y | 0.00156 |

$$\begin{aligned} \epsilon_y^{Actual} &= \epsilon_y^T + \nu \epsilon_x^T \\ &= 0.00156 \end{aligned}$$

Free Body Diagram Output (Controlling Factored Loadcase)

Controlling Analysis Load: BUCKLING

| | Nx, ex | Ny, ey | Nxy, yxy | Mx, ix | My, iy | Mxy, ixy | Qx | Qy |
|-----------------------|---------|--------------|----------|--------|--------|----------|----|----|
| Virtual Loads | -1200 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Design-to Loads | -1200 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Design-to Deformation | -0.0012 | 3.599999E-04 | 0 | 0 | 0 | 0 | | |

Ex 7 –Applied ΔT Partially Constrained



Input (Per Load Case)

ULTIMATE-THERMAL Load Case #1 "one" (Mechanical Set #101, Thermal Set #201)

Mechanical Load Set #101 "Load Set 101"
 Thermal Load Set #201 "Load Set 201"

FEA Loads - Projects Only
 User Loads Applied Unit Value

For Strength Analysis
 For Buckling Analysis

| | | | | | | | |
|-------------|--------|----------|--------|--------|----------|------|------|
| Ref Temp | 100 | Temp | 200 | | | | |
| Pressure | | TT Grad | 0 | | | | |
| Nx, ex | Ny, ey | Nxy, yxy | Mx, ix | My, iy | Mxy, ixy | Qx | Qy |
| Constrained | Free | Free | Free | Free | Free | Load | Load |

Thermal Strain

$$\begin{aligned} \epsilon_x^T &= \epsilon_y^T = \alpha \Delta T \\ &= (12 \times 10^{-6})(100) \\ &= 0.0012 \end{aligned}$$

Strain Actual (Computed Properties Tab)

| Deformation | |
|-------------|---------|
| Strain X | 0 |
| Strain Y | 0.00156 |

$$\begin{aligned} \epsilon_y^{Actual} &= \epsilon_y^T + \nu \epsilon_x^T \\ &= 0.00156 \end{aligned}$$

Design-To Strain

$$\begin{aligned} \epsilon_x^{Actual} &= \epsilon_x^T \\ \epsilon_x^{Design-To} &= \cancel{\epsilon_x^{Actual}} - \epsilon_x^T \\ &= -0.0012 \end{aligned}$$

$$\begin{aligned} \epsilon_y^{Design-To} &= \epsilon_y^{Actual} - \epsilon_y^T \\ \epsilon_y^{Design-To} &= (0.00156) - (0.0012) \\ &= 0.00036 \end{aligned}$$

Free Body Diagram Output (Controlling Factored Loadcase)

Controlling Analysis Load: BUCKLING

| | Nx, ex | Ny, ey | Nxy, yxy | Mx, ix | My, iy | Mxy, ixy | Qx | Qy |
|-----------------------|---------|--------------|----------|--------|--------|----------|----|----|
| Virtual Loads | -1200 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Design-to Loads | -1200 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Design-to Deformation | -0.0012 | 3.599999E-04 | 0 | 0 | 0 | 0 | | |

Ex 7 –Applied ΔT Partially Constrained



Input (Per Load Case)

ULTIMATE-THERMAL Load Case #1 "one" (Mechanical Set #101, Thermal Set #201)

Mechanical Load Set #101 "Load Set 101"
 Thermal Load Set #201 "Load Set 201"

FEA Loads - Projects Only
 User Loads Applied Unit Value

For Strength Analysis
 For Buckling Analysis

| | | | | | | | |
|-------------|--------|----------|--------|--------|----------|------|------|
| Nx, ex | Ny, ey | Nxy, yxy | Mx, ix | My, iy | Mxy, ixy | Qx | Qy |
| Constrained | Free | Free | Free | Free | Free | Load | Load |

Ref Temp: 100 Temp: 200
 Pressure: TT Grad: 0

Thermal Strain

$$\begin{aligned}\epsilon_x^T &= \epsilon_y^T = \alpha \Delta T \\ &= (12 \times 10^{-6})(100) \\ &= 0.0012\end{aligned}$$

Design-To Force

$$\begin{aligned}N_x^{Design-To} &= A_{11}\epsilon_x^{Design-To} + A_{12}\epsilon_y^{Design-To} \\ &= (1.0989 \times 10^6)(-0.0012) + (0.3)(1.0989 \times 10^6)(0.00036) \\ &= -1200\end{aligned}$$

Strain Actual (Computed Properties Tab)

| Deformation | |
|-------------|---------|
| Strain X | 0 |
| Strain Y | 0.00156 |

$$\begin{aligned}\epsilon_y^{Actual} &= \epsilon_y^T + \nu \epsilon_x^T \\ &= 0.00156\end{aligned}$$

Design-To Strain

$$\begin{aligned}\epsilon_x^{Actual} &= \epsilon_x^T \\ \epsilon_x^{Design-To} &= \cancel{\epsilon_x^{Actual}} - \epsilon_x^T \\ &= -0.0012\end{aligned}$$

$$\begin{aligned}\epsilon_y^{Design-To} &= \epsilon_y^{Actual} - \epsilon_y^T \\ \epsilon_y^{Design-To} &= (0.00156) - (0.0012) \\ &= 0.00036\end{aligned}$$

Free Body Diagram Output (Controlling Factored Loadcase)

Controlling Analysis Load: BUCKLING

| | Nx, ex | Ny, ey | Nxy, yxy | Mx, ix | My, iy | Mxy, ixy | Qx | Qy |
|-----------------------|---------|--------------|----------|--------|--------|----------|----|----|
| Virtual Loads | -1200 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Design-to Loads | -1200 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Design-to Deformation | -0.0012 | 3.599999E-04 | 0 | 0 | 0 | 0 | | |

Ex 8 – Thru-Thickness ΔT , Edges Free



Input (Per Load Case) $\Delta T = 1000$ F

ULTIMATE-THERMAL Load Case #1 "one" (Mechanical Set #101, Thermal Set #201)

Mechanical Load Set #101 "Load Set 101"
 Thermal Load Set #201 "Load Set 201"

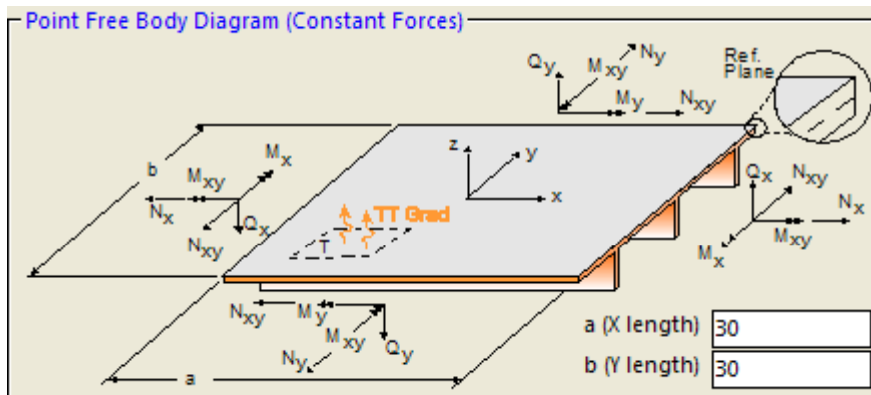
FEA Loads - Projects Only
 User Loads Applied Unit Value

For Strength Analysis
 For Buckling Analysis

| | Nx, ex | Ny, ey | Nxy, yxy | Mx, ex | My, ey | Mxy, xxy | Qx | Qy |
|--|--------|--------|----------|--------|--------|----------|------|------|
| | Free | Free | Free | Free | Free | Free | Load | Load |
| | | | | | | | | |
| | | | | | | | | |

Ref Temp: 100 Temp: 100
 Pressure: TT Grad: 1000

What will the strains look like?



Ex 8 – Thru-Thickness ΔT , Edges Free



Input (Per Load Case)

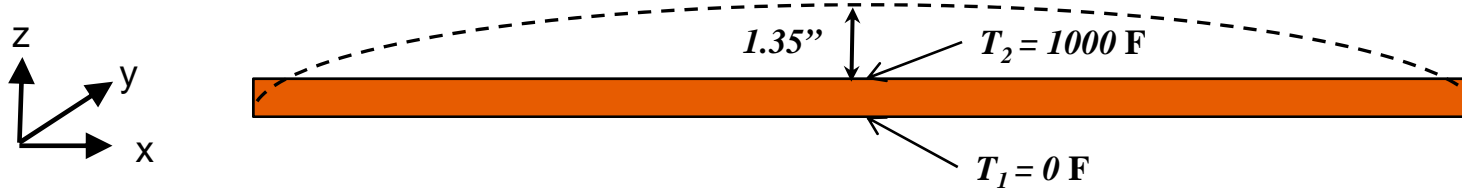
ULTIMATE-THERMAL Load Case #1 "one" (Mechanical Set #101, Thermal Set #201)

Mechanical Load Set #101 "Load Set 101"
 Thermal Load Set #201 "Load Set 201"
 FEA Loads - Projects Only
 User Loads

Applied Unit Value: For Strength Analysis, For Buckling Analysis

| | Nx,ex | Ny,ey | Nxy,xyy | Mx,xx | My,yy | Mxy,xyy | Qx | Qy |
|--|-------|-------|---------|-------|-------|---------|------|------|
| | Free | Free | Free | Free | Free | Free | Load | Load |
| | | | | | | | | |

Ref Temp: 100 Temp: 100
 Pressure: TT Grad: 1000



Deformation

| | | |
|--------------------|--|---------|
| Strain X | | -0.0006 |
| Strain Y | | -0.0006 |
| Curvature X | | -0.012 |
| Curvature Y | | -0.012 |
| Midspan Deflection | | 1.35 |

Free Body Diagram Output (Controlling Factored Loadcase)

Controlling Analysis Load: BUCKLING

| | Nx,ex | Ny,ey | Nxy,xyy | Mx,xx | My,yy | Mxy,xyy | Qx | Qy |
|-----------------------|-------|-------|---------|-------|-------|---------|----|----|
| Virtual Loads | | | | | | | | |
| Design-to Loads | | | | | | | | |
| Design-to Deformation | 0 | 0 | 0 | 0 | 0 | 0 | | |

Ex 9 – Panel Pressure



Input (Per Load Case)

LIMIT-MECHANICAL Load Case #1 "one" (Mechanical Set #101, Thermal Set #201)

Mechanical Load Set #101 "Load Set 101"
 Thermal Load Set #201 "Load Set 201"

FEA Loads - Projects Only
 User Loads Applied Unit Value

For Strength Analysis

For Buckling Analysis


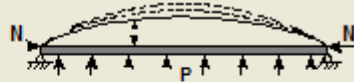
| | Nx,ex | Ny,ey | Nxy,xy | Mx,xx | My,xy | Mxy,xy | Qx | Qy |
|--|-------|-------|--------|-------|-------|--------|------|------|
| | Free | Free | Free | Free | Free | Free | Load | Load |
| | | | | | | | | |

Ref Temp Temp

Pressure TT Grad

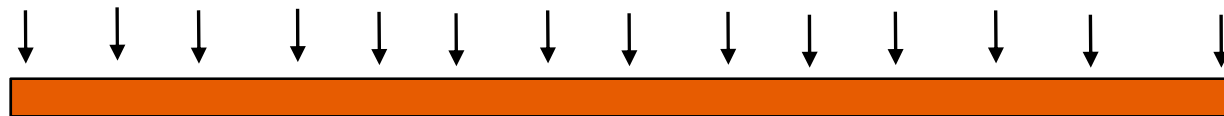
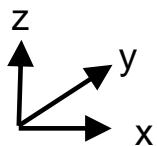
Superimposed Loads

Panel Pressure
 Beam-Column Moments

Initial Imperfection

Zero Out FEA Computed Moments



Ex 9 – Panel Pressure



Input (Per Load Case)

LIMIT-MECHANICAL Load Case #1 "one" (Mechanical Set #101, Thermal Set #201)

Mechanical Load Set #101 "Load Set 101"
 Thermal Load Set #201 "Load Set 201"

FEA Loads - Projects Only
 User Loads Applied Unit Value


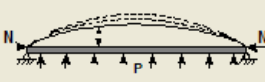
For Strength Analysis
 For Buckling Analysis

| | | | | | | | |
|-------|-------|---------|-------|-------|---------|------|------|
| Nx,εx | Ny,εy | Nxy,γxy | Mx,εx | My,εy | Mxy,εxy | Qx | Qy |
| Free | Free | Free | Free | Free | Free | Load | Load |
| | | | | | | | |
| | | | | | | | |

Ref Temp: Temp:
 Pressure: -100 TT Grad:

Superimposed Loads

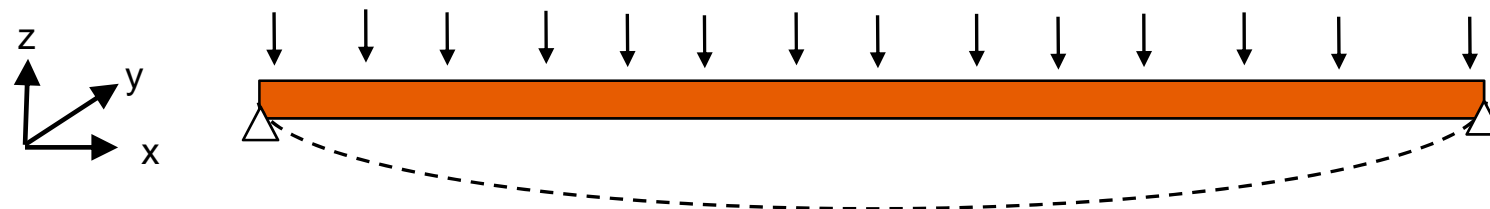
Panel Pressure
 Beam-Column Moments

Initial Imperfection:

Zero Out FEA Computed Moments
 SIMPLE Boundary Condition

| | Mx | My | Qx | Qy |
|----------|----------|----------|-----------|-----------|
| MidSpan | 4309.657 | 4309.657 | 0 | 0 |
| EdgeCntr | 0 | 0 | -997.5873 | -997.5873 |



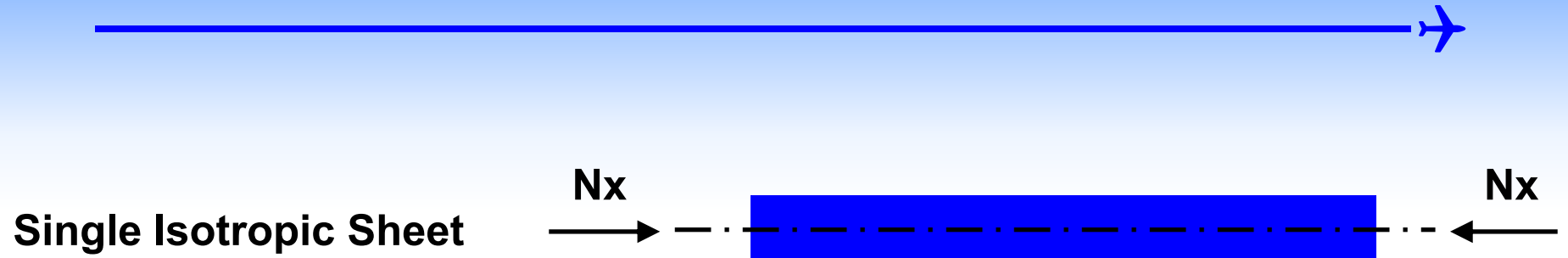
Free Body Diagram Output (Controlling Factored Loadcase)

Controlling Analysis Load: STRENGTH

| Virtual Loads | Nx,εx | Ny,εy | Nxy,γxy | Mx,εx | My,εy | Mxy,εxy | Qx | Qy |
|-----------------------|-------|-------|---------|----------|----------|---------|----|----|
| Design-to Loads | 0 | 0 | 0 | 4309.66 | 4309.66 | 0 | 0 | 0 |
| Design-to Deformation | 0 | 0 | 0 | 3.620112 | 3.620112 | 0 | | |

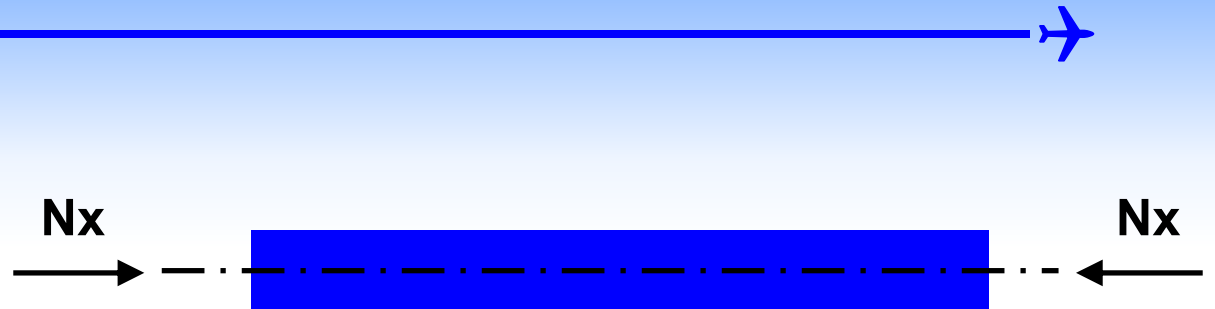


Panel Edge Loading



Panel Edge Loading

Single Isotropic Sheet



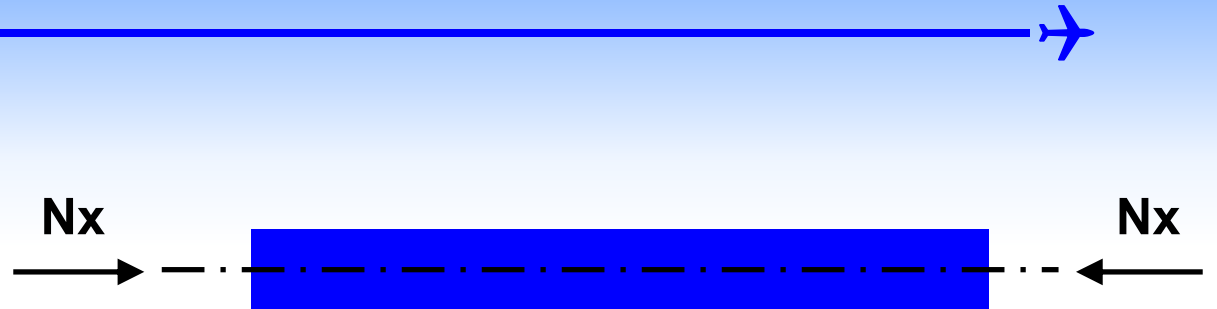
B Matrix = 0
No Membrane-Bending
Coupling

$$\begin{Bmatrix} \mathbf{N} \\ \mathbf{M} \end{Bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{0} & \mathbf{D} \end{bmatrix} \begin{Bmatrix} \boldsymbol{\varepsilon} \\ \boldsymbol{\kappa} \end{Bmatrix}$$



Panel Edge Loading

Single Isotropic Sheet



B Matrix = 0
No Membrane-Bending
Coupling

$$\begin{Bmatrix} \mathbf{N} \\ \mathbf{M} \end{Bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{0} & \mathbf{D} \end{bmatrix} \begin{Bmatrix} \boldsymbol{\varepsilon} \\ \boldsymbol{\kappa} \end{Bmatrix}$$

$$\mathbf{N} = \mathbf{A}\boldsymbol{\varepsilon}$$

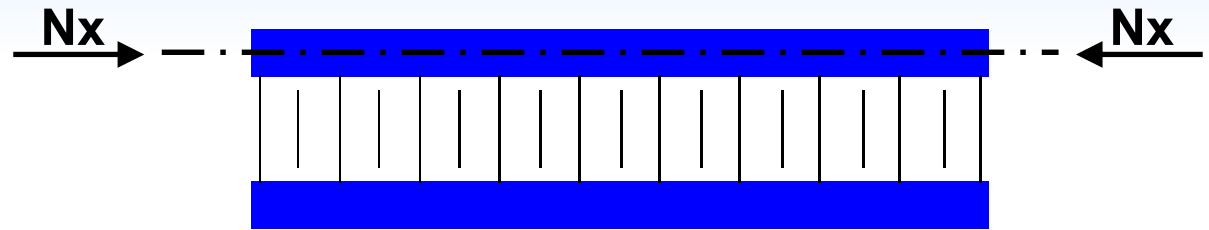
$$\mathbf{M} = \mathbf{D}\boldsymbol{\kappa}$$



Panel Edge Loading



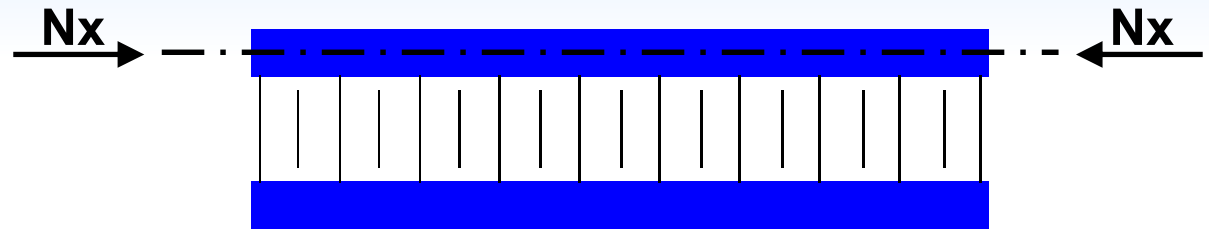
Symmetric Honeycomb Sandwich
(Note Reference Plane)



Panel Edge Loading



Symmetric Honeycomb Sandwich
(Note Reference Plane)



B Matrix $\neq 0$
Membrane-bending
Coupling is present

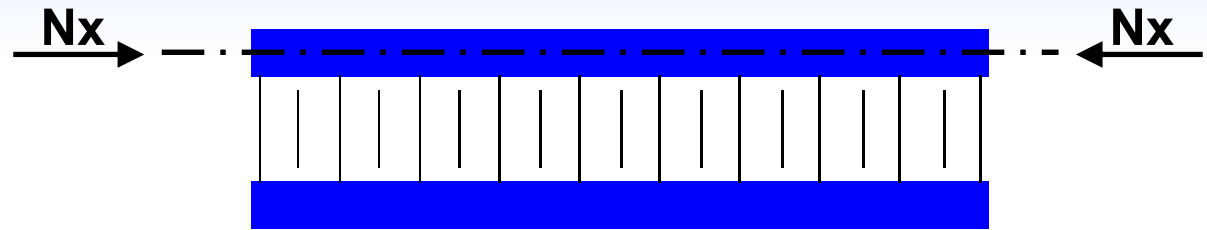
$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} \epsilon \\ \kappa \end{Bmatrix}$$



Panel Edge Loading



Symmetric Honeycomb Sandwich
(Note Reference Plane)



B Matrix $\neq 0$
Membrane-bending
Coupling is present

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} \epsilon \\ \kappa \end{Bmatrix}$$

$$N = A\epsilon + B\kappa$$

$$M = B\epsilon + D\kappa$$



Free Body Diagram Math



Calculated by
HyperSizer

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_{xy} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$$



Free Body Diagram Math



$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_{xy} \\ K_x \\ K_y \\ K_{xy} \end{Bmatrix}$$

Calculated by HyperSizer

“Knowns”



Free Body Diagram Math



“Unknowns”

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix}$$

=

Calculated by
HyperSizer

$$\begin{bmatrix} A & B \\ B & D \end{bmatrix}$$

“Knowns”

$$\begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_{xy} \\ K_x \\ K_y \\ K_{xy} \end{Bmatrix}$$

Free Body Diagram Math



“Unknowns”

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix}$$

=

Calculated by
HyperSizer

$$\begin{bmatrix} A & B \\ B & D \end{bmatrix}$$

“Knowns”

$$\begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_{xy} \\ K_x \\ K_y \\ K_{xy} \end{Bmatrix}$$

Unknowns on left, Knowns on right



Free Body Diagram Math – FEM Import



$$\left\{ \right\} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B} & \mathbf{D} \end{bmatrix} \left\{ \begin{array}{c} \mathbf{N}_x \\ \mathbf{N}_y \\ \mathbf{N}_{xy} \\ \mathbf{M}_x \\ \mathbf{M}_y \\ \mathbf{M}_{xy} \end{array} \right\}$$

FORCES



Free Body Diagram Math – FEM Import



STRAINS

$$\left\{ \begin{array}{c} \epsilon_x \\ \epsilon_y \\ \epsilon_{xy} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{array} \right\} =$$

$$\left[\begin{array}{cc} \mathbf{A} & \mathbf{B} \\ \mathbf{B} & \mathbf{D} \end{array} \right]$$

FORCES

$$\left\{ \begin{array}{c} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{array} \right\}$$



Free Body Diagram Math – FEM Import



STRAINS

$$\begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_{xy} \\ K_x \\ K_y \\ K_{xy} \end{Bmatrix} =$$

Inverted Matrix

$$\begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B} & \mathbf{D} \end{bmatrix}^{-1}$$

FORCES

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix}$$

Free Body Diagram Math – FEM Import



STRAINS

$$\begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_{xy} \\ K_x \\ K_y \\ K_{xy} \end{Bmatrix}$$

=

Inverted Matrix

$$\begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B} & \mathbf{D} \end{bmatrix}^{-1}$$

FORCES

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix}$$

$$\epsilon_x = A^{-1}_{11} N_x + A^{-1}_{12} N_y + \dots$$



Free Body Diagram Math – FEM Import



STRAINS

$$\begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_{xy} \\ K_x \\ K_y \\ K_{xy} \end{Bmatrix} =$$

Inverted Matrix

$$\begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B} & \mathbf{D} \end{bmatrix}^{-1}$$

6x6

FORCES

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix}$$

$$\epsilon_x = A^{-1}_{11} N_x + A^{-1}_{12} N_y + \dots$$



Free Body Diagram Math – FEM Import



$$\begin{matrix} \text{STRAINS} \\ \left\{ \begin{array}{c} \epsilon_x \\ \epsilon_y \\ \epsilon_{xy} \\ K_x \\ K_y \\ K_{xy} \end{array} \right\} \end{matrix} = \begin{matrix} \text{Inverted Matrix} \\ \left[\begin{array}{cc} \mathbf{A} & \mathbf{B} \\ \mathbf{B} & \mathbf{D} \end{array} \right]^{-1} \\ \text{6x6} \end{matrix} \begin{matrix} \text{FORCES} \\ \left\{ \begin{array}{c} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{array} \right\} \end{matrix}$$



Free Body Diagram Math – FEM Import



$$\begin{array}{c} \mathbf{STRAINS} \\ \left\{ \begin{array}{c} \epsilon_x \\ \epsilon_y \\ \epsilon_{xy} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{array} \right\} = \left[\begin{array}{cc} \mathbf{A} & \mathbf{B} \\ \mathbf{B} & \mathbf{D} \end{array} \right]^{-1} \left\{ \begin{array}{c} \mathbf{FORCES} \\ N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{array} \right\} \end{array}$$

6x6

When coupling HyperSizer with a FEM, the FEA computed forces are imported to compute panel strains and curvatures this way. (At the reference plane)



FBD Math – Workspace Loads



Specified Strain

| | N_x, ϵ_x | N_y, ϵ_y | N_{xy}, γ_{xy} | M_x, κ_x | M_y, κ_y | M_{xy}, κ_{xy} |
|-----------------------|-------------------|-------------------|-----------------------|-----------------|-----------------|-----------------------|
| Applied Unit Value | Deformation ▾ | Constrained ▾ | Constrained ▾ | Constrained ▾ | Constrained ▾ | Constrained ▾ |
| For Strength Analysis | 0.01 | | | | | |
| For Buckling Analysis | 0.01 | | | | | |

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_{xy} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$$



FBD Math – Workspace Loads



Specified Strain

| | N_x, ϵ_x | N_y, ϵ_y | N_{xy}, γ_{xy} | M_x, κ_x | M_y, κ_y | M_{xy}, κ_{xy} |
|-----------------------|-------------------|-------------------|-----------------------|-----------------|-----------------|-----------------------|
| Applied Unit Value | Deformation | Constrained | Constrained | Constrained | Constrained | Constrained |
| For Strength Analysis | 0.01 | | | | | |
| For Buckling Analysis | 0.01 | | | | | |

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} \epsilon_y \\ \epsilon_{xy} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$$

The value 0.01 in the table above is circled in orange and has an arrow pointing to the top element of the strain vector on the right, which is also circled in orange.



FBD Math – Workspace Loads

Specified Strain



| | N_x, ϵ_x | N_y, ϵ_y | N_{xy}, γ_{xy} | M_x, α_x | M_y, α_y | M_{xy}, α_{xy} |
|-----------------------|-------------------|-------------------|-----------------------|-----------------|-----------------|-----------------------|
| Applied Unit Value | Deformation | Constrained | Constrained | Constrained | Constrained | Constrained |
| For Strength Analysis | 0.01 | | | | | |
| For Buckling Analysis | 0.01 | | | | | |

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} 0.01 \\ 0.0 \\ 0.0 \\ 0.0 \\ 0.0 \\ 0.0 \end{Bmatrix}$$



FBD Math – Workspace Loads

Specified Strain



| | N_x, ϵ_x | N_y, ϵ_y | N_{xy}, γ_{xy} | M_x, κ_x | M_y, κ_y | M_{xy}, κ_{xy} |
|-----------------------|-------------------|-------------------|-----------------------|-----------------|-----------------|-----------------------|
| Applied Unit Value | Deformation | Constrained | Constrained | Constrained | Constrained | Constrained |
| For Strength Analysis | 0.01 | | | | | |
| For Buckling Analysis | 0.01 | | | | | |

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} 0.01 \\ 0.0 \\ 0.0 \\ 0.0 \\ 0.0 \\ 0.0 \end{Bmatrix}$$

$$N_x = A_{11} \epsilon_x + A_{12} + 0.0$$

$$N_y = A_{21} \epsilon_x + A_{22} + 0.0$$



FBD Math – Workspace Loads



Specified Load

| | Nx, ex | Ny, ey | Nxy, γxy | Mx, rx | My, ry | Mxy, rxy |
|-----------------------|--------|---------------|---------------|---------------|---------------|---------------|
| Applied Unit Value | Load ▾ | Constrained ▾ | Constrained ▾ | Constrained ▾ | Constrained ▾ | Constrained ▾ |
| For Strength Analysis | -100 | | | | | |
| For Buckling Analysis | -100 | | | | | |

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_{xy} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$$



FBD Math – Workspace Loads

Specified Load

| | N_x, ϵ_x | N_y, ϵ_y | N_{xy}, γ_{xy} | M_x, κ_x | M_y, κ_y | M_{xy}, κ_{xy} |
|-----------------------|-------------------|-------------------|-----------------------|-----------------|-----------------|-----------------------|
| Applied Unit Value | Load | Constrained | Constrained | Constrained | Constrained | Constrained |
| For Strength Analysis | -100 | | | | | |
| For Buckling Analysis | -100 | | | | | |

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_{xy} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$$

N_x is now known, ϵ_x is unknown



FBD Math – Workspace Loads

Specified Load

| | N_x, ϵ_x | N_y, ϵ_y | N_{xy}, γ_{xy} | M_x, κ_x | M_y, κ_y | M_{xy}, κ_{xy} |
|-----------------------|-------------------|-------------------|-----------------------|-----------------|-----------------|-----------------------|
| Applied Unit Value | Load | Constrained | Constrained | Constrained | Constrained | Constrained |
| For Strength Analysis | -100 | | | | | |
| For Buckling Analysis | -100 | | | | | |

$$\begin{Bmatrix} \epsilon_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A' & B' \\ B' & D' \end{bmatrix} \begin{Bmatrix} N_x \\ \epsilon_y \\ \epsilon_{xy} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$$

Switch N_x and ϵ_x - Rearrange ABD



FBD Math – Workspace Loads



Specified Load

| | N_x, ϵ_x | N_y, ϵ_y | N_{xy}, γ_{xy} | M_x, ϵ_x | M_y, ϵ_y | M_{xy}, ϵ_{xy} |
|-----------------------|-------------------|-------------------|-----------------------|-------------------|-------------------|-------------------------|
| Applied Unit Value | Load | Constrained | Constrained | Constrained | Constrained | Constrained |
| For Strength Analysis | -100 | | | | | |
| For Buckling Analysis | -100 | | | | | |

$$\begin{Bmatrix} \epsilon_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A' & B' \\ B' & D' \end{bmatrix} \begin{Bmatrix} -100 \\ 0.0 \\ 0.0 \\ 0.0 \\ 0.0 \\ 0.0 \end{Bmatrix}$$



FBD Math – Workspace Loads



Virtual Loads

| | Nx, ex | Ny, ey | Nxy, yxy | Mx, ex | My, ey | Mxy, exy |
|-----------------------|--------|-------------|-------------|-------------|-------------|-------------|
| Applied Unit Value | Load | Constrained | Constrained | Constrained | Constrained | Constrained |
| For Strength Analysis | -100 | | | | | |
| For Buckling Analysis | -100 | | | | | |

$$\begin{Bmatrix} \epsilon_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A' & B' \\ B' & D' \end{bmatrix} \begin{Bmatrix} N_x \\ \epsilon_y \\ \epsilon_{xy} \\ K_x \\ K_y \\ K_{xy} \end{Bmatrix}$$

| Controlling Analysis Load: STRENGTH | Nx, ex | Ny, ey | Nxy, yxy | Mx, ex | My, ey | Mxy, exy |
|-------------------------------------|---------------|--------|----------|--------|--------|----------|
| Virtual Loads | 0 | -31 | 0 | 0 | 0 | 0 |
| Design-to Loads | -100 | -31 | 0 | 0 | 0 | 0 |
| Design-to Deformation | -4.351252E-05 | 0 | 0 | 0 | 0 | 0 |



FBD Math – Workspace Loads



Virtual Loads

| | Nx, ex | Ny, ey | Nxy, yxy | Mx, ex | My, ey | Mxy, exy |
|-----------------------|--------|-------------|-------------|-------------|-------------|-------------|
| Applied Unit Value | Load | Constrained | Constrained | Constrained | Constrained | Constrained |
| For Strength Analysis | -100 | | | | | |
| For Buckling Analysis | -100 | | | | | |

$$\begin{Bmatrix} \epsilon_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A' & B' \\ B' & D' \end{bmatrix} \begin{Bmatrix} N_x \\ \epsilon_y \\ \epsilon_{xy} \\ K_x \\ K_y \\ K_{xy} \end{Bmatrix}$$

| Controlling Analysis Load: STRENGTH | Nx, ex | Ny, ey | Nxy, yxy | Mx, ex | My, ey | Mxy, exy |
|-------------------------------------|---------------|--------|----------|--------|--------|----------|
| Virtual Loads | 0 | -31 | 0 | 0 | 0 | 0 |
| Design-to Loads | -100 | -31 | 0 | 0 | 0 | 0 |
| Design-to Deformation | -4.351252E-05 | | 0 | 0 | 0 | 0 |



FBD Math – Workspace Loads



Virtual Loads

| | Nx, ϵ_x | Ny, ϵ_y | Nxy, γ_{xy} | Mx, κ_x | My, κ_y | Mxy, κ_{xy} |
|-----------------------|------------------|------------------|--------------------|----------------|----------------|--------------------|
| Applied Unit Value | Load | Constrained | Constrained | Constrained | Constrained | Constrained |
| For Strength Analysis | -100 | | | | | |
| For Buckling Analysis | -100 | | | | | |

$$\begin{Bmatrix} \epsilon_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A' & B' \\ B' & D' \end{bmatrix} \begin{Bmatrix} N_x \\ \epsilon_y \\ \epsilon_{xy} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$$

| Controlling Analysis Load: STRENGTH | Nx, ϵ_x | Ny, ϵ_y | Nxy, γ_{xy} | Mx, κ_x | My, κ_y | Mxy, κ_{xy} |
|-------------------------------------|------------------|------------------|--------------------|----------------|----------------|--------------------|
| Virtual Loads | 0 | -31 | 0 | 0 | 0 | 0 |
| Design-to Loads | -100 | -31 | 0 | 0 | 0 | 0 |
| Design-to Deformation | -4.351252E-05 | | 0 | 0 | 0 | 0 |

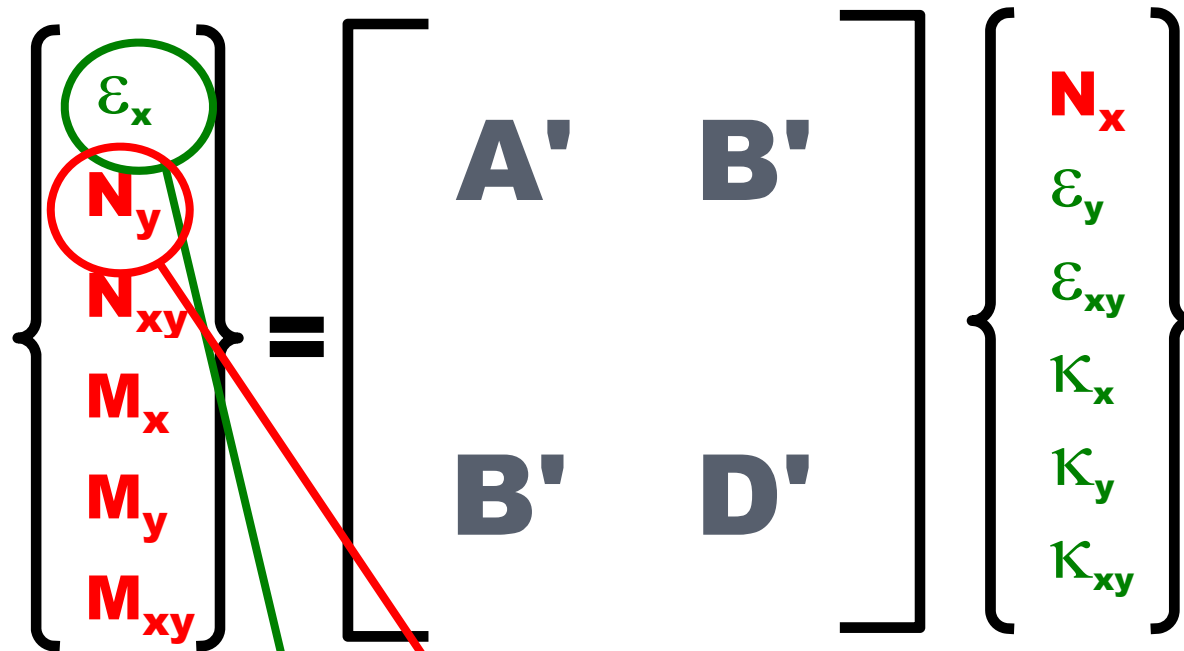


FBD Math – Workspace Loads



Virtual Loads

$$N_y = A'_{21} N_x + A'_{22} \epsilon_y + \dots$$



| Controlling Analysis Load: STRENGTH | Nx, ex | Ny, ey | Nxy, yxy | Mx, ex | My, ey | Mxy, exy |
|-------------------------------------|---------------|--------|----------|--------|--------|----------|
| Virtual Loads | 0 | -31 | 0 | 0 | 0 | 0 |
| Design-to Loads | -100 | -31 | 0 | 0 | 0 | 0 |
| Design-to Deformation | -4.351252E-05 | | 0 | 0 | 0 | 0 |



Free Boundary Conditions



| Applied Unit Value | N_x, ϵ_x | N_y, ϵ_y | N_{xy}, γ_{xy} | M_x, κ_x | M_y, κ_y | M_{xy}, κ_{xy} |
|-----------------------|-------------------|-------------------|-----------------------|-----------------|-----------------|-----------------------|
| For Strength Analysis | Load | Free | Constrained | Constrained | Constrained | Constrained |
| For Buckling Analysis | -100 | | | | | |
| | -100 | | | | | |

$$\begin{Bmatrix} \epsilon_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A' & B' \\ B' & D' \end{bmatrix} \begin{Bmatrix} N_x \\ \epsilon_y \\ \epsilon_{xy} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$$

| Controlling Analysis Load: STRENGTH | N_x, ϵ_x | N_y, ϵ_y | N_{xy}, γ_{xy} | M_x, κ_x | M_y, κ_y | M_{xy}, κ_{xy} |
|-------------------------------------|-------------------|-------------------|-----------------------|-----------------|-----------------|-----------------------|
| Virtual Loads | | | | | | |
| Design-to Loads | -100 | 0 | 0 | 0 | 0 | 0 |
| Design-to Deformation | -5.212941E-05 | 1.616012E-05 | 0 | 0 | 0 | 0 |



Free Boundary Conditions



| Applied Unit Value | N_x, ϵ_x | N_y, ϵ_y | N_{xy}, γ_{xy} | M_x, κ_x | M_y, κ_y | M_{xy}, κ_{xy} |
|-----------------------|-------------------|-------------------|-----------------------|-----------------|-----------------|-----------------------|
| For Strength Analysis | Load | Free | Constrained | Constrained | Constrained | Constrained |
| For Buckling Analysis | -100 | | | | | |

$$\begin{Bmatrix} \epsilon_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A' & B' \\ B' & D' \end{bmatrix} \begin{Bmatrix} N_x \\ \epsilon_y \\ \epsilon_{xy} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$$

| Controlling Analysis Load: STRENGTH | N_x, ϵ_x | N_y, ϵ_y | N_{xy}, γ_{xy} | M_x, κ_x | M_y, κ_y | M_{xy}, κ_{xy} |
|-------------------------------------|-------------------|-------------------|-----------------------|-----------------|-----------------|-----------------------|
| Virtual Loads | | | | | | |
| Design-to Loads | -100 | 0 | 0 | 0 | 0 | 0 |
| Design-to Deformation | -5.212941E-05 | 1.616012E-05 | 0 | 0 | 0 | 0 |



Free Boundary Conditions



| Applied Unit Value | N_x, ϵ_x | N_y, ϵ_y | N_{xy}, γ_{xy} | M_x, κ_x | M_y, κ_y | M_{xy}, κ_{xy} |
|-----------------------|-------------------|-------------------|-----------------------|-----------------|-----------------|-----------------------|
| Load | ▼ | Free ▼ | Constrained ▼ | Constrained ▼ | Constrained ▼ | Constrained ▼ |
| For Strength Analysis | -100 | | | | | |
| For Buckling Analysis | -100 | | | | | |

$$\begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A'' & B'' \\ B'' & D'' \end{bmatrix} \begin{Bmatrix} N_x \\ N_y = 0 \\ \epsilon_{xy} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$$

The diagram shows a matrix equation where the left-hand side vector contains strain and stress components, and the right-hand side vector contains stress and curvature components. A red arrow points from the $N_y = 0$ entry in the right-hand side vector to the N_y, ϵ_y entry in the top table, indicating the boundary condition.

| Controlling Analysis Load: STRENGTH | N_x, ϵ_x | N_y, ϵ_y | N_{xy}, γ_{xy} | M_x, κ_x | M_y, κ_y | M_{xy}, κ_{xy} |
|-------------------------------------|-------------------|-------------------|-----------------------|-----------------|-----------------|-----------------------|
| Virtual Loads | | | | | | |
| Design-to Loads | -100 | 0 | 0 | 0 | 0 | 0 |
| Design-to Deformation | -5.212941E-05 | 1.616012E-05 | 0 | 0 | 0 | 0 |



Free Boundary Conditions



| Applied Unit Value | N_x, ϵ_x | N_y, ϵ_y | N_{xy}, γ_{xy} | M_x, κ_x | M_y, κ_y | M_{xy}, κ_{xy} |
|-----------------------|-------------------|-------------------|-----------------------|-----------------|-----------------|-----------------------|
| Load | Free | Free | Constrained | Constrained | Constrained | Constrained |
| For Strength Analysis | -100 | | | | | |
| For Buckling Analysis | -100 | | | | | |

$$\begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A'' & B'' \\ B'' & D'' \end{bmatrix} \begin{Bmatrix} N_x \\ N_y \\ \epsilon_{xy} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} = 0$$

| Controlling Analysis Load: STRENGTH | N_x, ϵ_x | N_y, ϵ_y | N_{xy}, γ_{xy} | M_x, κ_x | M_y, κ_y | M_{xy}, κ_{xy} |
|-------------------------------------|-------------------|-------------------|-----------------------|-----------------|-----------------|-----------------------|
| Virtual Loads | | | | | | |
| Design-to Loads | -100 | 0 | 0 | 0 | 0 | 0 |
| Design-to Deformation | -5.212941E-05 | 1.616012E-05 | 0 | 0 | 0 | 0 |



Free Boundary Conditions



| Applied Unit Value | N_x, ϵ_x | N_y, ϵ_y | N_{xy}, γ_{xy} | M_x, κ_x | M_y, κ_y | M_{xy}, κ_{xy} |
|-----------------------|-------------------|-------------------|-----------------------|-----------------|-----------------|-----------------------|
| For Strength Analysis | Load | Free | Constrained | Constrained | Constrained | Constrained |
| For Buckling Analysis | -100 | | | | | |
| | -100 | | | | | |

$$\begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A'' & B'' \\ B'' & D'' \end{bmatrix} \begin{Bmatrix} N_x \\ N_y \\ \epsilon_{xy} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} = 0$$

Note: double prime 6x6 matrix

| Controlling Analysis Load: STRENGTH | N_x, ϵ_x | N_y, ϵ_y | N_{xy}, γ_{xy} | M_x, κ_x | M_y, κ_y | M_{xy}, κ_{xy} |
|-------------------------------------|-------------------|-------------------|-----------------------|-----------------|-----------------|-----------------------|
| Virtual Loads | | | | | | |
| Design-to Loads | -100 | 0 | 0 | 0 | 0 | 0 |
| Design-to Deformation | -5.212941E-05 | 1.616012E-05 | 0 | 0 | 0 | 0 |



Analyses are Independent of Loads Source



HyperSizer Failure Analyses



Analyses are Independent of Loads Source



User Input by hand, (typed-in loads).
Very convenient interactive tool

| Variables | | FBD | | Object Lc |
|--|-------|-------|--------|-------------|
| Input (Per Load Case) | | | | |
| **ULTIMATE-MECHANICAL** Load Case #1 "one" (Mechanical Set #101, Thermal Set #201) | | | | |
| <input checked="" type="radio"/> Mechanical Load Set #101 "Load Set 101" | | | | |
| <input type="radio"/> Thermal Load Set #201 "Load Set 201" | | | | |
| <input type="radio"/> FEA Loads - Projects Only | | | | |
| <input checked="" type="radio"/> User Loads | | | | |
| Applied Unit Value | Nx,ex | Ny,ey | Nxy,xy | |
| For Strength Analysis | -2000 | | Load | Constrained |
| For Buckling Analysis | -2000 | | | Deformation |



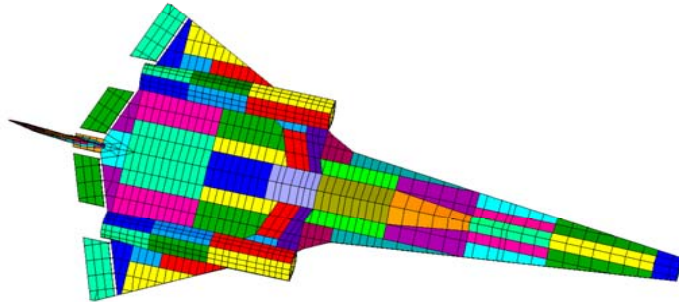
HyperSizer Failure Analyses



Analyses are Independent of Loads Source



FEA Computed Loads



User Input by hand, (typed-in loads).
Very convenient interactive tool

| Variables | FBD | Object Lc | |
|--|-------|-----------|---------|
| Input (Per Load Case) | | | |
| **ULTIMATE-MECHANICAL** Load Case #1 "one" (Mechanical Set #101, Thermal Set #201) | | | |
| <input checked="" type="radio"/> Mechanical Load Set #101 "Load Set 101" | | | |
| <input type="radio"/> Thermal Load Set #201 "Load Set 201" | | | |
| <input type="radio"/> FEA Loads - Projects Only | | | |
| <input checked="" type="radio"/> User Loads | | | |
| Applied Unit Value | Nx,ex | Ny,ey | Nxy,xy |
| For Strength Analysis | -2000 | | 0.00042 |
| For Buckling Analysis | -2000 | | 0.00036 |

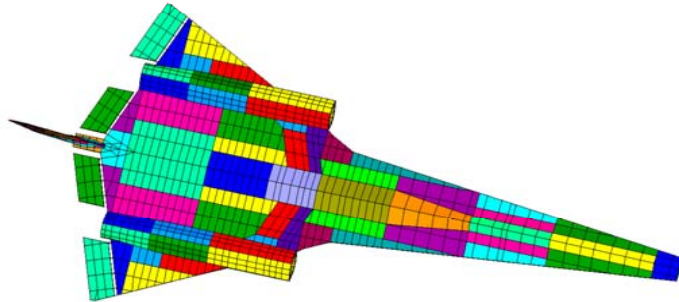
HyperSizer Failure Analyses



Analyses are Independent of Loads Source



FEA Computed Loads



User Input by hand, (typed-in loads).
Very convenient interactive tool

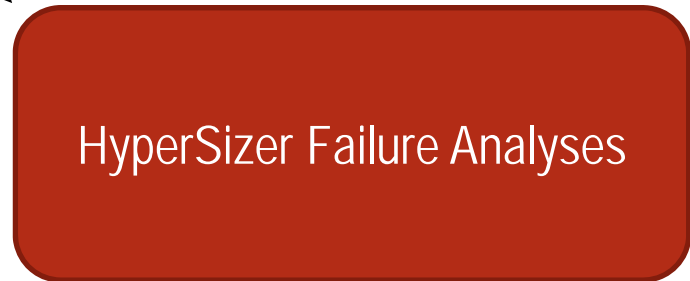
| Variables | FBD | Object Lc | |
|--|-------|-------------|-------------|
| Input (Per Load Case) | | | |
| **ULTIMATE-MECHANICAL** Load Case #1 "one" (Mechanical Set #101, Thermal Set #201) | | | |
| <input checked="" type="radio"/> Mechanical Load Set #101 "Load Set 101" | | | |
| <input type="radio"/> Thermal Load Set #201 "Load Set 201" | | | |
| <input type="radio"/> FEA Loads - Projects Only | | | |
| <input checked="" type="radio"/> User Loads | | | |
| Applied Unit Value | Nx,ex | Ny,ey | Nxy,xy |
| For Strength Analysis | Load | Constrained | Deformation |
| For Buckling Analysis | -2000 | | 0.00042 |
| | -2000 | | 0.00036 |

Other Sources Using HyperSizer's Object Model Interface:

- Loads from spreadsheets



- Loads from a larger company software design system





Appendix

Appendix I: ABD of Isotropic Plate



- Reduced stiffness matrix Q
 - Plane stress constitutive equation
 - In-plane properties E , ν , & G

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} \frac{E}{1-\nu^2} & \frac{\nu E}{1-\nu^2} & 0 \\ \frac{\nu E}{1-\nu^2} & \frac{E}{1-\nu^2} & 0 \\ 0 & 0 & G_{12} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} = \mathbf{Q} \vec{\varepsilon}$$

Appendix I: ABD of Isotropic Plate



- Integrate Q over the single layer
 - $h_0 = -t/2, h_1 = t/2$

$$A_{ij} = \sum_{k=1}^n Q_{ij}(\theta_k)(h_k - h_{k-1}) = \boxed{Qt}$$

$$B_{ij} = \sum_{k=1}^n Q_{ij}(\theta_k)(h_k^2 - h_{k-1}^2)/2 = Q\left(\frac{t^2}{4} - \frac{t^2}{4}\right)/2 = \boxed{0}$$

$$D_{ij} = \sum_{k=1}^n Q_{ij}(\theta_k)(h_k^3 - h_{k-1}^3)/3 = Q\left(\frac{t^3}{8} - \frac{t^3}{8}\right)/3 = \boxed{Q\frac{t^3}{12}}$$

Appendix I: ABD of Isotropic Plate



- Final ABD

$$\begin{bmatrix} N_x \\ N_y \\ N_s \\ M_x \\ M_y \\ M_s \end{bmatrix} = \begin{bmatrix} \frac{Et}{1-\nu^2} & \frac{\nu Et}{1-\nu^2} & 0 & 0 & 0 & 0 \\ \frac{\nu Et}{1-\nu^2} & \frac{Et}{1-\nu^2} & 0 & 0 & 0 & 0 \\ 0 & 0 & G_{12}t & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{Et^3}{12(1-\nu^2)} & \frac{\nu Et^3}{12(1-\nu^2)} & 0 \\ 0 & 0 & 0 & \frac{\nu Et^3}{12(1-\nu^2)} & \frac{Et^3}{12(1-\nu^2)} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{G_{12}t^3}{12} \end{bmatrix} \begin{bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \\ \kappa_x \\ \kappa_y \\ \kappa_z \end{bmatrix}$$

Appendix II: Iso Effective Elastic Constants



- Goal is reduce ABD relation of isotropic plate to the forms:
 - N_x load, N_y free, N_{xy} free, M free

$$\varepsilon_x = \frac{N_x}{tE_x^{eff}}$$

$$\varepsilon_y = -\nu_{xy}^{eff} \varepsilon_x$$

Appendix II: Iso Effective Elastic Constants



- Start with isotropic ABD

$$\begin{bmatrix} N_x \\ N_y \\ N_s \\ M_x \\ M_y \\ M_s \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 & 0 & 0 & 0 \\ A_{12} & A_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & A_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & D_{11} & D_{12} & 0 \\ 0 & 0 & 0 & D_{12} & D_{22} & 0 \\ 0 & 0 & 0 & 0 & 0 & D_{33} \end{bmatrix} \begin{bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \\ \kappa_x \\ \kappa_y \\ \kappa_z \end{bmatrix}$$

Appendix II: Iso Effective Elastic Constants



- Invert

$$\begin{bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \\ \kappa_x \\ \kappa_y \\ \kappa_z \end{bmatrix} = \begin{bmatrix} \frac{A_{22}}{A_{11}A_{22} - A_{12}^2} & \frac{-A_{12}}{A_{11}A_{22} - A_{12}^2} & 0 & 0 & 0 & 0 \\ \frac{-A_{12}}{A_{11}A_{22} - A_{12}^2} & \frac{A_{11}}{A_{11}A_{22} - A_{12}^2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{A_{33}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{D_{22}}{-D_{12}^2 + D_{11}D_{22}} & \frac{-D_{12}}{-D_{12}^2 + D_{11}D_{22}} & 0 \\ 0 & 0 & 0 & \frac{-D_{12}}{-D_{12}^2 + D_{11}D_{22}} & \frac{D_{11}}{-D_{12}^2 + D_{11}D_{22}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{D_{33}} \end{bmatrix} \begin{bmatrix} N_x \\ N_y \\ N_s \\ M_x \\ M_y \\ M_s \end{bmatrix}$$

Appendix II: Iso Effective Elastic Constants



- N_x load, N_y free, N_{xy} free, M free

$$\begin{aligned}\epsilon_x^o &= \frac{A_{22}}{A_{11}A_{22} - A_{12}^2} N_x \\ &= \frac{N_x}{E_x^{eff}}\end{aligned}$$

$$\begin{aligned}\epsilon_y^o &= \frac{-A_{12}}{A_{11}A_{22} - A_{12}^2} N_x \\ &= \frac{-A_{12}}{A_{11}A_{22} - A_{12}^2} \frac{A_{11}A_{22} - A_{12}^2}{A_{22}} \epsilon_x^o \\ &= \frac{-A_{12}}{A_{22}} \epsilon_x^o \\ &= -\nu_{xy}^{eff} \epsilon_x^o\end{aligned}$$

Appendix III: ABD⁻¹ of Isotropic Plate



- Inverted ABD – see Appendix II

$$\begin{bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \\ \kappa_x \\ \kappa_y \\ \kappa_z \end{bmatrix} = \begin{bmatrix} \frac{A_{22}}{A_{11}A_{22} - A_{12}^2} & \frac{-A_{12}}{A_{11}A_{22} - A_{12}^2} & 0 & 0 & 0 & 0 \\ \frac{-A_{12}}{A_{11}A_{22} - A_{12}^2} & \frac{A_{11}}{A_{11}A_{22} - A_{12}^2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{A_{33}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{D_{22}}{-D_{12}^2 + D_{11}D_{22}} & \frac{-D_{12}}{-D_{12}^2 + D_{11}D_{22}} & 0 \\ 0 & 0 & 0 & \frac{-D_{12}}{-D_{12}^2 + D_{11}D_{22}} & \frac{D_{11}}{-D_{12}^2 + D_{11}D_{22}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{D_{33}} \end{bmatrix} \begin{bmatrix} N_x \\ N_y \\ N_s \\ M_x \\ M_y \\ M_s \end{bmatrix}$$

Appendix III: ABD⁻¹ of Isotropic Plate



- Simplify inverse matrix terms

$$A_{11}^{-1} = \frac{A_{22}}{A_{11}A_{22} - A_{12}^2} = \frac{Et}{(Et)^2 - (Etv)^2} = \frac{1 - \nu^2}{Et(1 - \nu^2)} = \frac{1}{Et}$$

$$A_{22}^{-1} = \frac{1}{Et}$$

$$A_{12}^{-1} = \frac{-\nu}{Et}$$

$$A_{33}^{-1} = \frac{1}{Gt}$$

$$\begin{bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \\ \kappa_x \\ \kappa_y \\ \kappa_z \end{bmatrix} = \begin{bmatrix} A_{11}^{-1} & A_{12}^{-1} & 0 & 0 & 0 & 0 \\ A_{12}^{-1} & A_{22}^{-1} & 0 & 0 & 0 & 0 \\ 0 & 0 & A_{33}^{-1} & 0 & 0 & 0 \\ 0 & 0 & 0 & D_{11}^{-1} & D_{12}^{-1} & 0 \\ 0 & 0 & 0 & D_{12}^{-1} & D_{22}^{-1} & 0 \\ 0 & 0 & 0 & 0 & 0 & D_{33}^{-1} \end{bmatrix} \begin{bmatrix} N_x \\ N_y \\ N_s \\ M_x \\ M_y \\ M_s \end{bmatrix}$$





Extra

Isotropic Plate Stiffness



□ Compliance

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} \frac{1}{E} & \frac{-\nu}{E} & 0 \\ \frac{-\nu}{E} & \frac{1}{E} & 0 \\ 0 & 0 & \frac{1}{G_{12}} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix}$$

Isotropic Plate Stiffness



□ Compliance

□ Stiffness

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} \frac{1}{E} & \frac{-\nu}{E} & 0 \\ \frac{-\nu}{E} & \frac{1}{E} & 0 \\ 0 & 0 & \frac{1}{G_{12}} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix}$$



$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} \frac{E}{1-\nu^2} & \frac{\nu E}{1-\nu^2} & 0 \\ \frac{\nu E}{1-\nu^2} & \frac{E}{1-\nu^2} & 0 \\ 0 & 0 & G_{12} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix}$$

Isotropic Plate Stiffness



□ Compliance

□ Stiffness

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} \frac{1}{E} & \frac{-\nu}{E} & 0 \\ \frac{-\nu}{E} & \frac{1}{E} & 0 \\ 0 & 0 & \frac{1}{G_{12}} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix}$$



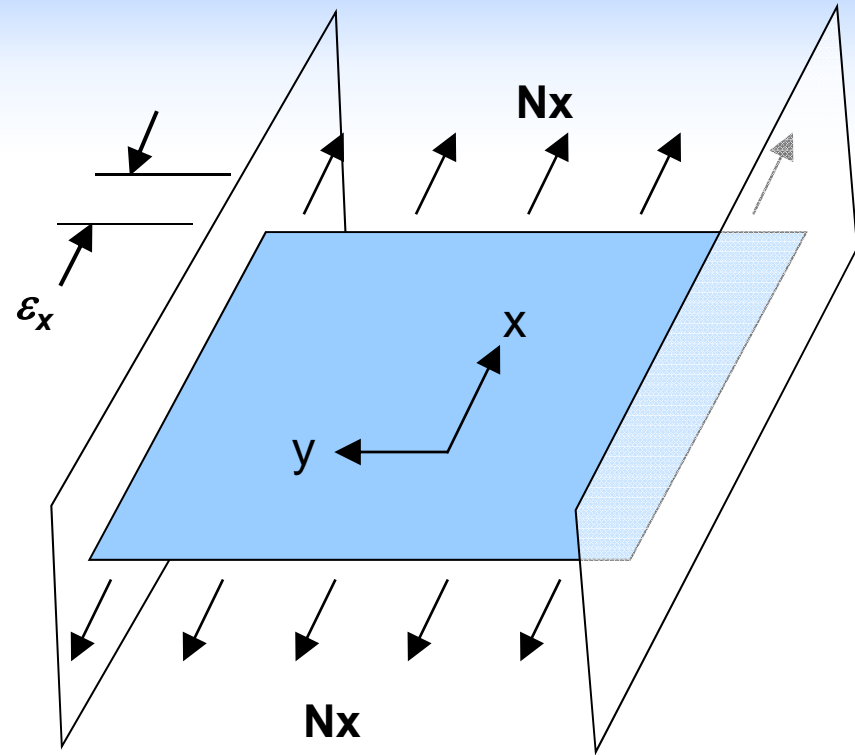
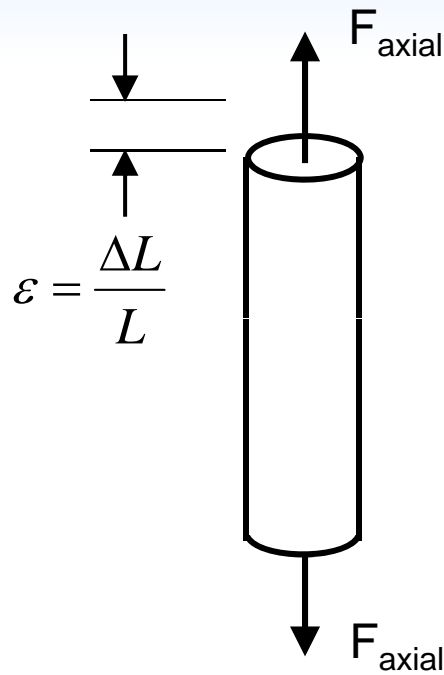
$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} \frac{E}{1-\nu^2} & \frac{\nu E}{1-\nu^2} & 0 \\ \frac{\nu E}{1-\nu^2} & \frac{E}{1-\nu^2} & 0 \\ 0 & 0 & G_{12} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix}$$

Poisson term
for plates

$$\frac{1}{1-\nu^2}$$



Effective Modulus ($\epsilon_y = 0$)



$$\sigma = E \epsilon$$

$$\sigma_x = \left(\frac{E}{1-\nu^2} \right) \epsilon_x + \cancel{\nu \left(\frac{E}{1-\nu^2} \right) \epsilon_y}$$

$$\sigma_x = \left(\frac{E}{1-\nu^2} \right) \epsilon_x = E^* \epsilon_x$$

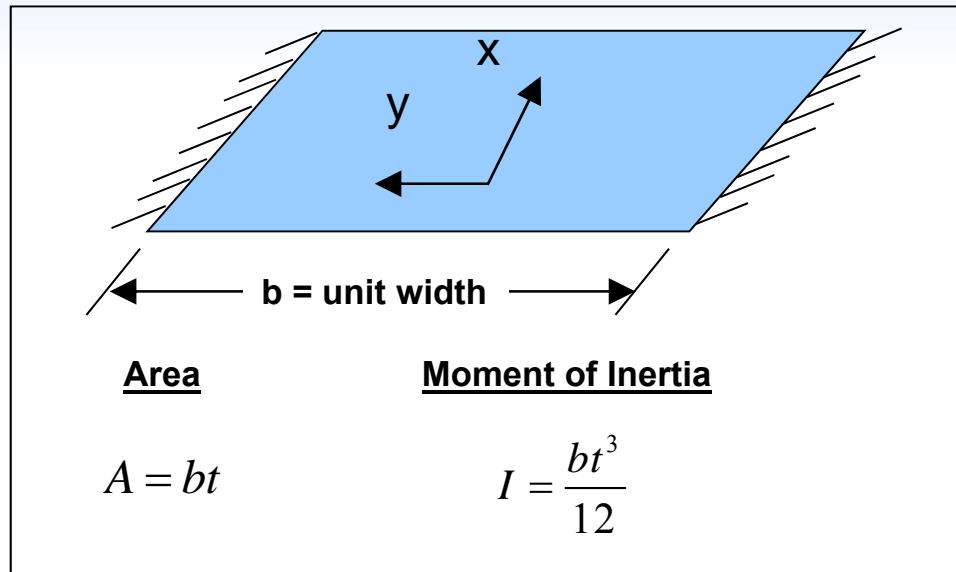


Convert Stress → Line Load



$$N_x = \sigma_x t$$

$$\sigma_x = E^* \varepsilon_x$$



$$M_x' = M_x t$$

$$M_x = E^* I \kappa$$

Membrane

Unit Force

$$\begin{aligned} N_x^* &= \left(\frac{E}{1-\nu^2} \right) \varepsilon t \\ &= \frac{Et}{1-\nu^2} \varepsilon_x \\ &= A_{11} \varepsilon_x \end{aligned}$$

Stiffness

$$A_{11} = \frac{Et}{1-\nu^2}$$

Bending

Unit Moment

$$\begin{aligned} M_x' &= \left(\frac{E}{1-\nu^2} \right) I \kappa t \\ &= \frac{Et^3}{12(1-\nu^2)} \kappa \\ &= D_{11} \kappa \end{aligned}$$

Bending Stiffness

$$D_{11} = \frac{Et^3}{12(1-\nu^2)}$$



Membrane Coupling Relationships



$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \begin{bmatrix} \frac{Et}{1-\nu^2} & \frac{\nu Et}{1-\nu^2} & 0 \\ \frac{\nu Et}{1-\nu^2} & \frac{Et}{1-\nu^2} & 0 \\ 0 & 0 & Gt \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} \quad \left| \quad \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \frac{1}{Et} & -\frac{\nu}{Et} & 0 \\ -\frac{\nu}{Et} & \frac{1}{Et} & 0 \\ 0 & 0 & \frac{1}{Gt} \end{bmatrix} \begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix}$$