1021_HyperSizer-Methods-Approach-FBD Tab Matrix Math.ppt

Collier Research Corporation
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Objective

- 1^{st} glimpse of HyperSizer Basic
- Get familiar with ABD matrix computations

Objective

- 1st glimpse of HyperSizer Basic
- Get familiar with ABD matrix computations
- Why?
  - HyperSizer smeared stiffness formulation

Outline

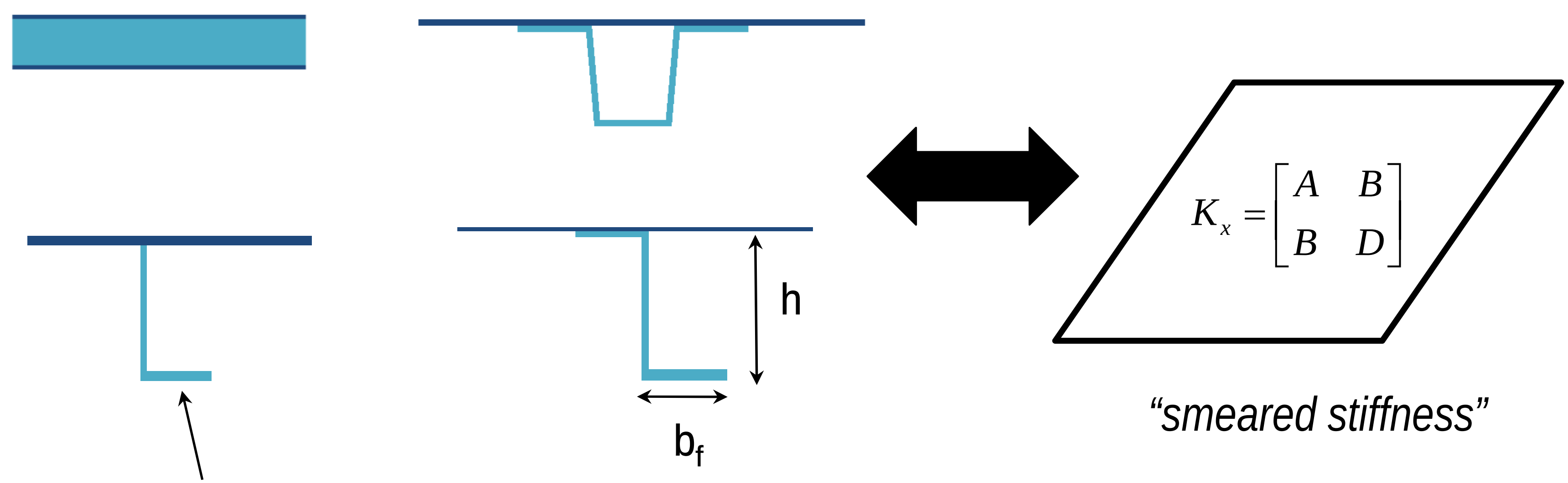
- HyperSizer panel stiffness approach
- Isotropic plate stiffness relations
- Metallic sheet examples
  - Mechanical loads
  - Thermal loads
  - Superimposed pressure

Panel Stiffness Approach

Panel Stiffness - Technical Approach

Local Stiffness

Global Stiffness



"smeared stiffness"

[45/90/90/-45/0/0/90/0]s

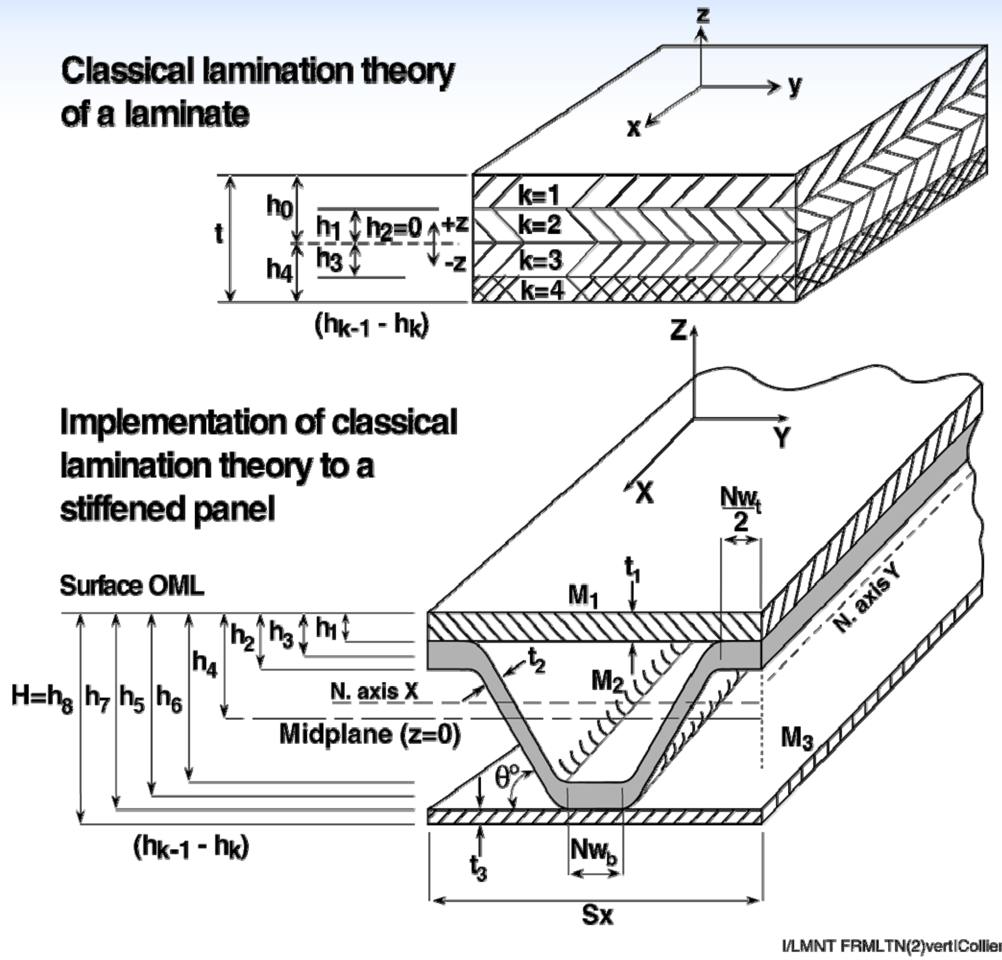
Panel Stiffness – Technical Approach

- Stiffened panel constitutive equation
  - [A] → membrane
  - [D] → bending
  - [B] → membrane-bending coupling

$$\begin{bmatrix} \vec{N} \\ \vec{M} \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B} & \mathbf{D} \end{bmatrix} \begin{bmatrix} \vec{\varepsilon} \\ \vec{\kappa} \end{bmatrix} - \begin{bmatrix} \vec{N}^T \\ \vec{M}^T \end{bmatrix}$$

Panel Stiffness - Technical Approach

- Classical Lamination Theory extended to a represent any stiffened cross sectional shape

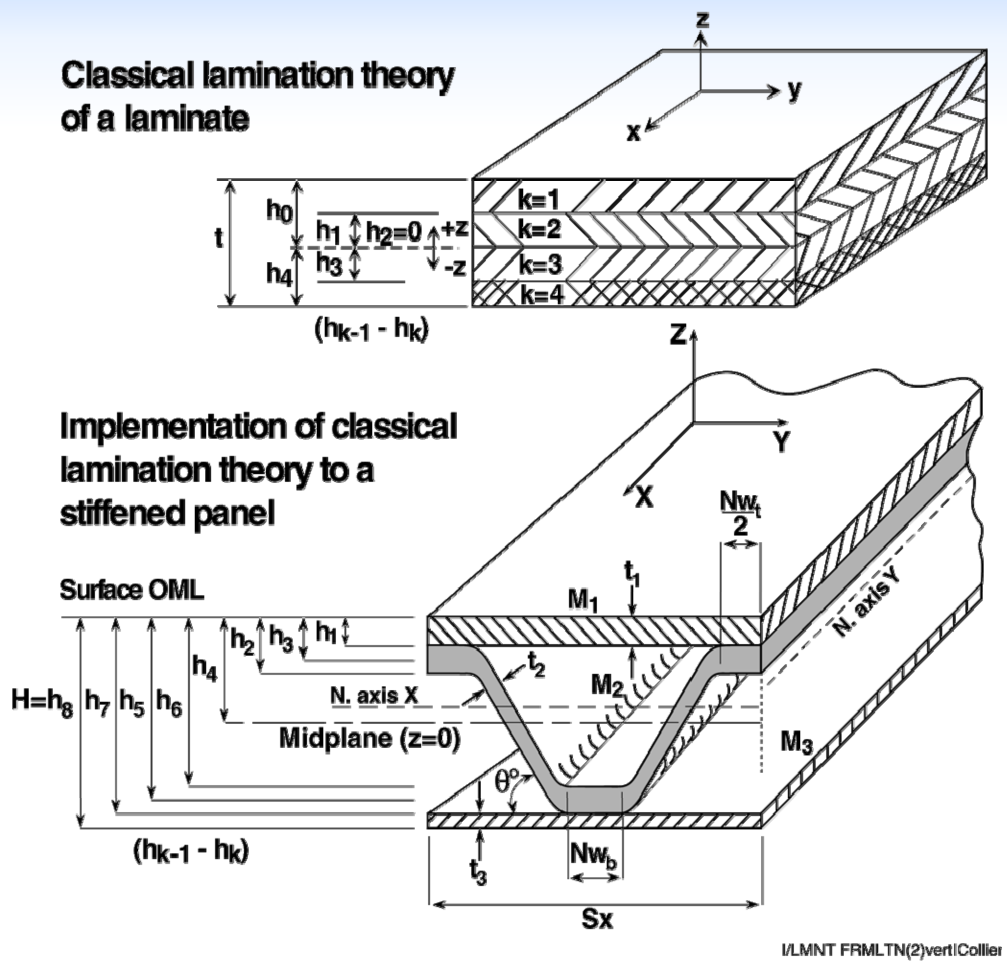


HyperSizer®

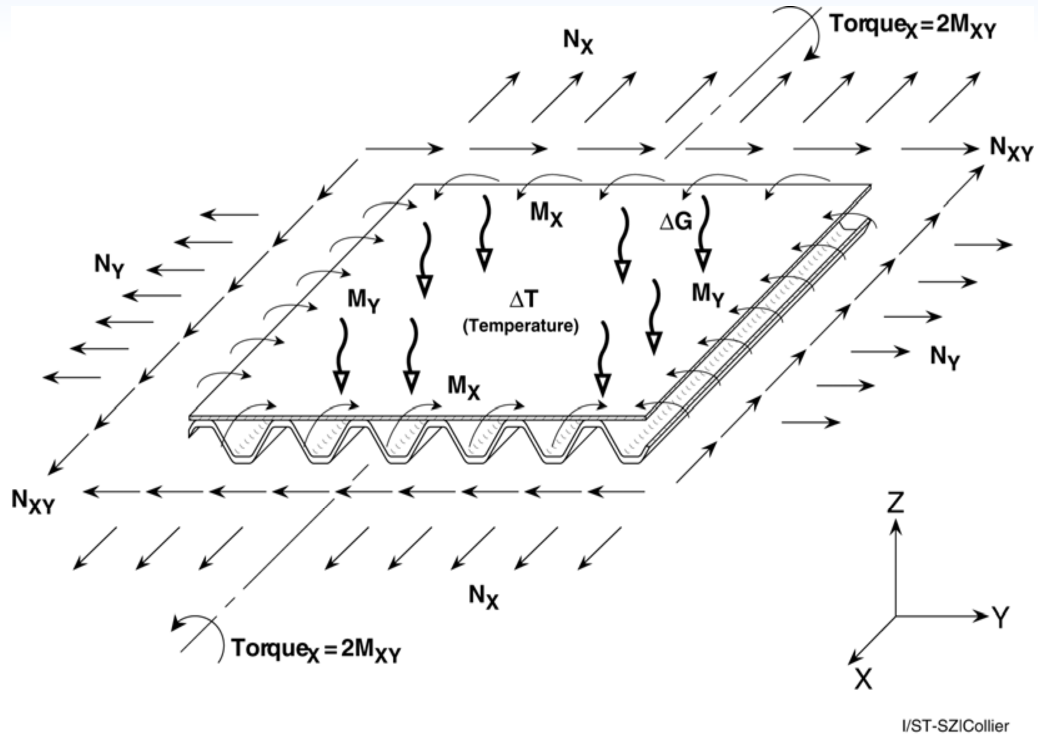
Panel Stiffness - Technical Approach

- **Classical Lamination Theory extended to a represent any stiffened cross sectional shape**

- **General panel behaviors, are quantified with:**
  - **Stiffeness terms [A], [B], [D]**
  - **Thermal coefficients [A^α], [B^α], [D^α]**

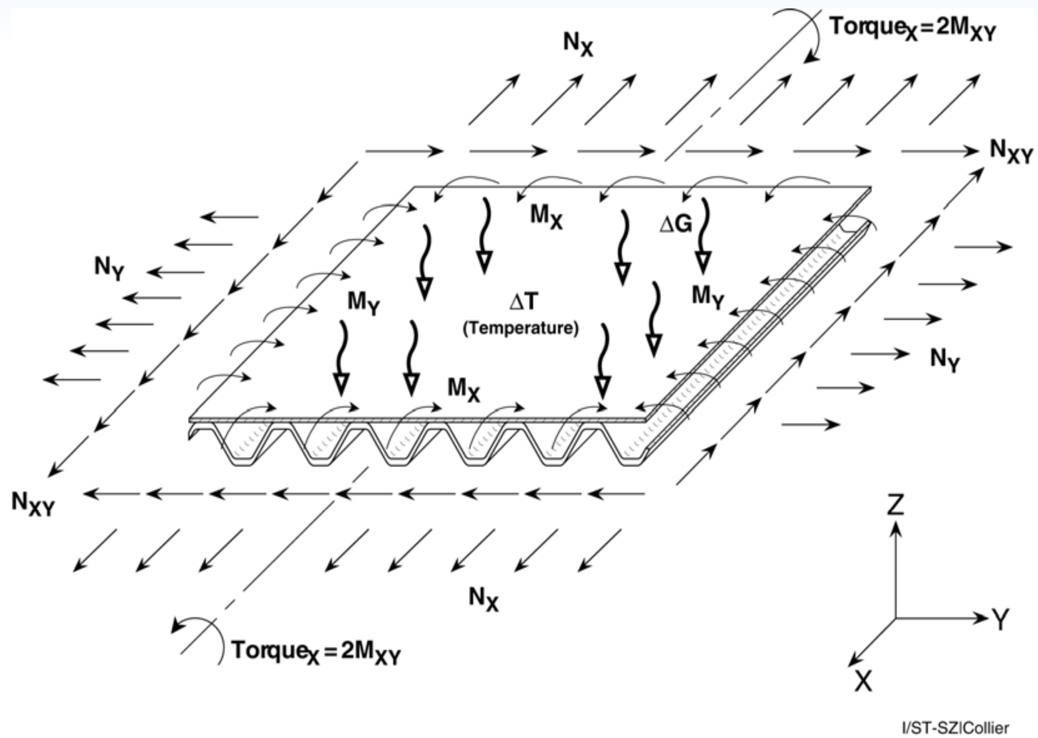


Free Body Diagram (FBD)



Free Body Diagram (FBD)

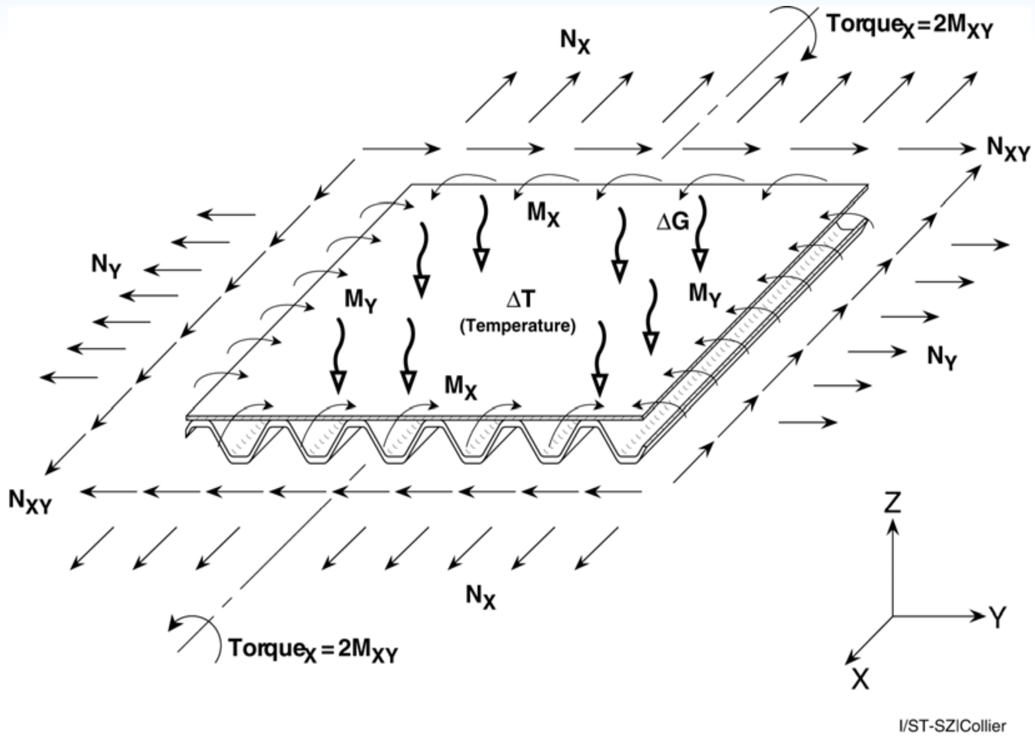
- **Balanced free-body loads**
  - **FEA**
  - **User-defined input**



Thermoelastic Formulations

Free Body Diagram (FBD)

- Balanced free-body loads
  - FEA
  - User-defined input
- Consistently applied thermoelastic formulations guarantee
  - Equilibrium of forces



Isotropic Plate Stiffness Relations

Isotropic Plate Stiffness

Plane Stress Constitutive Equations

- Compliance

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} \frac{1}{E} & \frac{-\nu}{E} & 0 \\ \frac{-\nu}{E} & \frac{1}{E} & 0 \\ 0 & 0 & \frac{1}{G_{12}} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix}$$

Isotropic Plate Stiffness

Plane Stress Constitutive Equations

- Compliance

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} \dfrac{1}{E} & \dfrac{-\nu}{E} & 0 \\ \dfrac{-\nu}{E} & \dfrac{1}{E} & 0 \\ 0 & 0 & \dfrac{1}{G_{12}} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix}$$

$$\Longleftrightarrow$$

- Stiffness

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} \dfrac{E}{1-\nu^2} & \dfrac{\nu E}{1-\nu^2} & 0 \\ \dfrac{\nu E}{1-\nu^2} & \dfrac{E}{1-\nu^2} & 0 \\ 0 & 0 & G_{12} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix}$$

Isotropic Plate Stiffness

Plane Stress Constitutive Equations

- Compliance

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} \frac{1}{E} & \frac{-\nu}{E} & 0 \\ \frac{-\nu}{E} & \frac{1}{E} & 0 \\ 0 & 0 & \frac{1}{G_{12}} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix}$$

- Stiffness

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} \frac{E}{1-\nu^2} & \frac{\nu E}{1-\nu^2} & 0 \\ \frac{\nu E}{1-\nu^2} & \frac{E}{1-\nu^2} & 0 \\ 0 & 0 & G_{12} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix}$$

Poisson term for plates $\frac{1}{1-\nu^2}$

ABD Matrix of Isotropic Plate

$$\begin{bmatrix} N_x \\ N_y \\ N_s \\ M_x \\ M_y \\ M_s \end{bmatrix} = \begin{bmatrix} \frac{Et}{1-\nu^2} & \frac{\nu Et}{1-\nu^2} & 0 & 0 & 0 & 0 \\ \frac{\nu Et}{1-\nu^2} & \frac{Et}{1-\nu^2} & 0 & 0 & 0 & 0 \\ 0 & 0 & Gt & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{Et^3}{12\left(1-\nu^2\right)} & \frac{\nu Et^3}{12\left(1-\nu^2\right)} & 0 \\ 0 & 0 & 0 & \frac{\nu Et^3}{12\left(1-\nu^2\right)} & \frac{Et^3}{12\left(1-\nu^2\right)} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{Gt^3}{12} \end{bmatrix} \begin{bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \\ \kappa_x \\ \kappa_y \\ \kappa_z \end{bmatrix}$$

ABD Matrix of Isotropic Plate

$$
\begin{bmatrix} N_x \\ N_y \\ N_s \\ M_x \\ M_y \\ M_s \end{bmatrix}
=
\begin{bmatrix}
\dfrac{Et}{1-\nu^2} & \dfrac{\nu Et}{1-\nu^2} & 0 & 0 & 0 & 0 \\
\dfrac{\nu Et}{1-\nu^2} & \dfrac{Et}{1-\nu^2} & 0 & 0 & 0 & 0 \\
0 & 0 & Gt & 0 & 0 & 0 \\
0 & 0 & 0 & \dfrac{Et^3}{12\left(1-\nu^2\right)} & \dfrac{\nu Et^3}{12\left(1-\nu^2\right)} & 0 \\
0 & 0 & 0 & \dfrac{\nu Et^3}{12\left(1-\nu^2\right)} & \dfrac{Et^3}{12\left(1-\nu^2\right)} & 0 \\
0 & 0 & 0 & 0 & 0 & \dfrac{Gt^3}{12}
\end{bmatrix}
\begin{bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \\ \kappa_x \\ \kappa_y \\ \kappa_z \end{bmatrix}
$$

ABD Matrix of Isotropic Plate

Membrane

$$A_{11} = A_{22} = \frac{Et}{1-\nu^2}$$

$$A_{12} = A_{21} = A_{11}\nu$$

$$A_{11}^{-1} = A_{22}^{-1} = \frac{1}{Et}$$

$$A_{12}^{-1} = A_{21}^{-1} = -A_{11}^{-1}\nu$$

Bending

$$D_{11} = D_{22} = \frac{Et^3}{12\left(1-\nu^2\right)}$$

$$D_{12} = D_{21} = D_{11}\nu$$

$$D_{11}^{-1} = D_{22}^{-1} = \frac{12}{Et^3}$$

$$D_{12}^{-1} = D_{21}^{-1} = -D_{11}^{-1}\nu$$

Examples

Isotropic Plate Force and Moment Exercises

Perform several exercises to verify the physics of the ABD matrix in using HyperSizer's operation user-defined loads

- Specified Strain
- Specified Force
- Uniform ΔT
- Through-Thickness ΔT
- Pressure

Set up for Demo Problem

<u>Copy an Isotropic: 'AL 7075' and rename to 'FBD Example'</u>

Set properties

$E_c = E_t = 10$ Msi

G = 3.846154 Msi

$\nu = 0.3$

$\alpha = 12^{-6}$

<u>Setup Group sizing bounds thickness equal to .1"</u>

<u>Set Ultimate Load Factor = 1.0</u>

Examples with Isotropic Plates

Isotropic Plate

$E = 10$ Msi
$\nu = 0.3$
$G = 3.846154$ Msi
Plate Thickness, $t = 0.1''$
CTE, $\alpha = 12\ \mu in/in$

Membrane

$$A_{11} = \frac{Et}{1-\nu^2} = \frac{(10)(0.1)}{0.91}$$

$$\boxed{= 1.0989 \times 10^6 \text{ lb/in}}$$

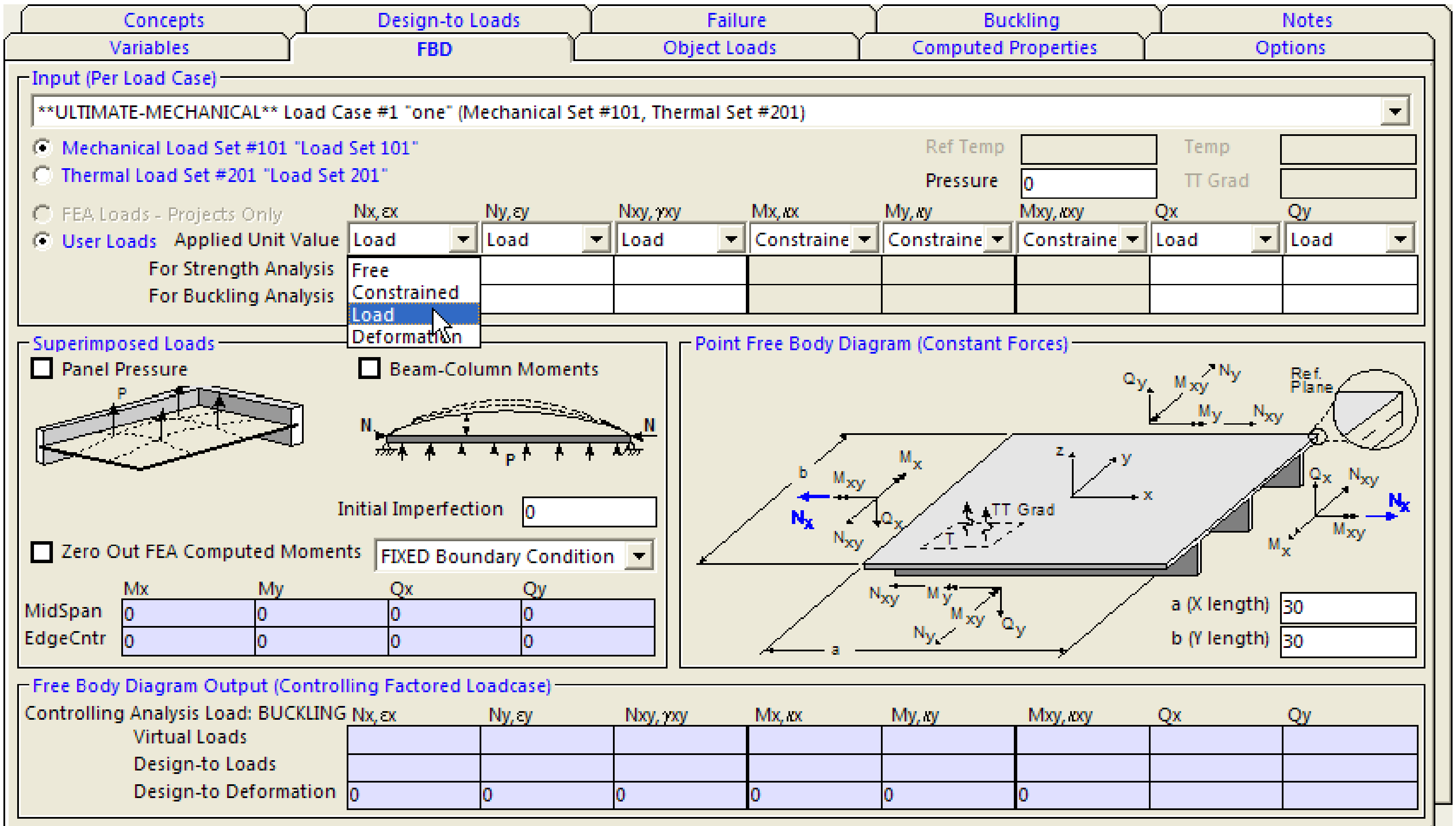
Bending

$$D_{11} = \frac{Et^3}{12(1-\nu^2)} = \frac{(10)(0.1)^3}{12(0.91)}$$

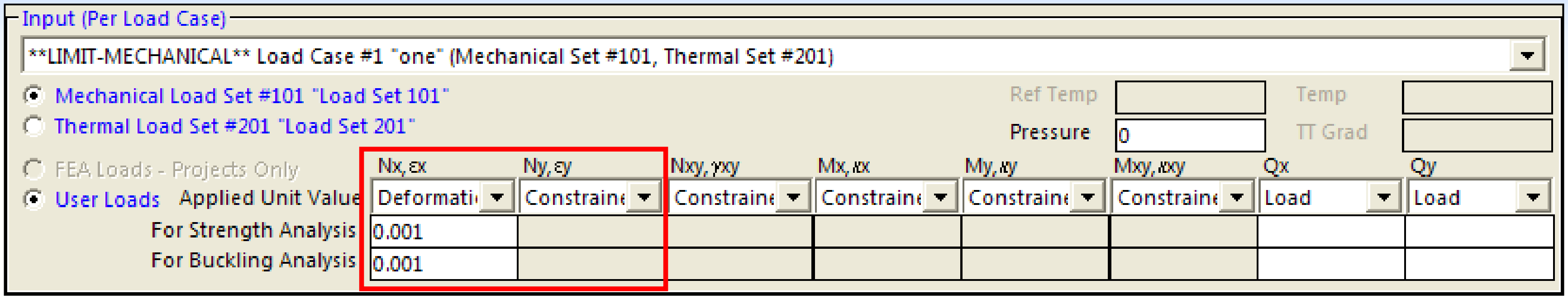
$$\boxed{= 915.8 \text{ lb/in}}$$

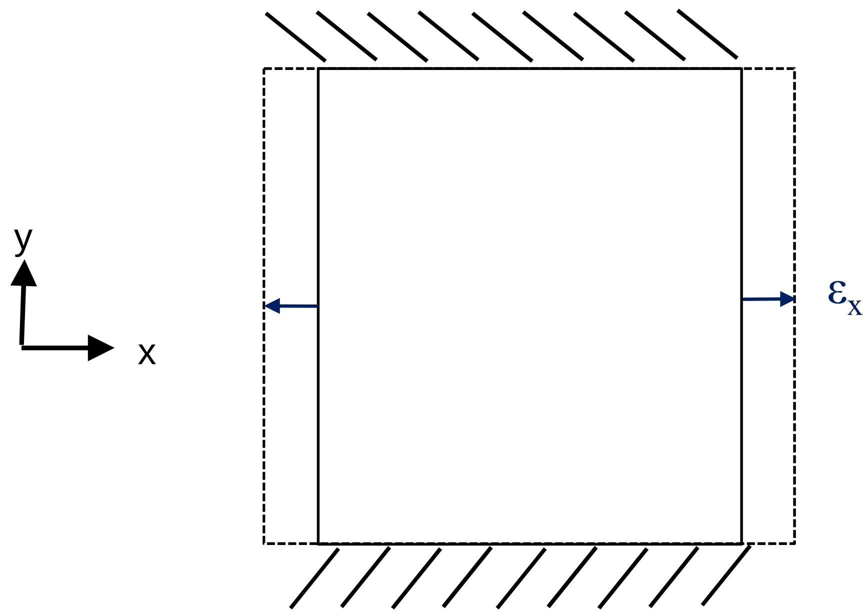
The Free Body Diagram Tab

- Entry of Loads and Boundary Conditions



Ex 1 – Applied ε_x, Constrained ε_y





What will the loads look like?

Positive, negative or zero?

Ex 1 – Applied ε_x, Constrained ε_y

Input (Per Load Case)

LIMIT-MECHANICAL Load Case #1 "one" (Mechanical Set #101, Thermal Set #201)

- (•) Mechanical Load Set #101 "Load Set 101"
- ( ) Thermal Load Set #201 "Load Set 201"

Ref Temp | Temp | Pressure: 0 | TT Grad

- ( ) FEA Loads - Projects Only
- (•) User Loads

	Nx,εx	Ny,εy	Nxy,γxy	Mx,κx	My,κy	Mxy,κxy	Qx	Qy
Applied Unit Value	Deformati	Constraine	Constraine	Constraine	Constraine	Constraine	Load	Load
For Strength Analysis	0.001							
For Buckling Analysis	0.001							

$$Nx = A_{11}\varepsilon_x + A_{12}\cancel{\varepsilon_y}$$
$$= \left(1.0989\times 10^6\right)\left(0.001\right)$$
$$\boxed{= 1098.9}$$

Set Ultimate Factor = 1.0

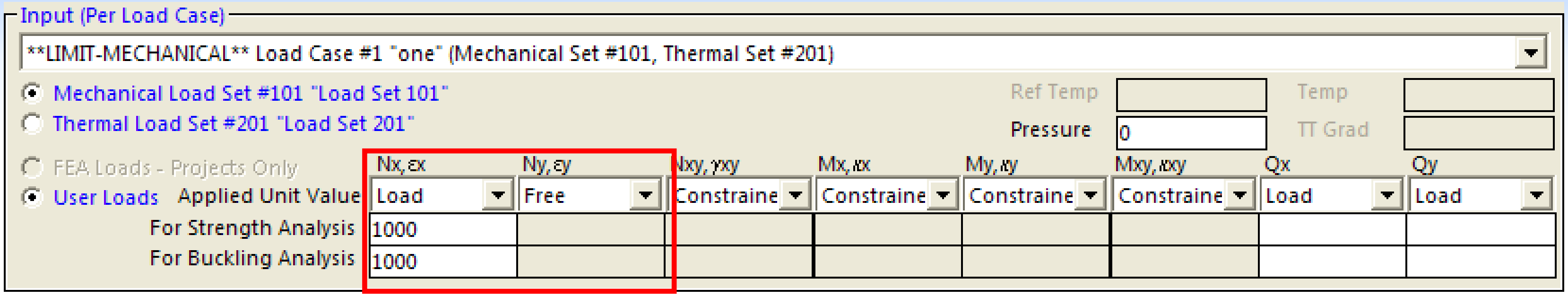
$$Ny = A_{21}\varepsilon_x = \nu A_{11}\varepsilon_x$$
$$= (0.3)\left(1.0989\times 10^6\right)\left(0.001\right)$$
$$\boxed{= 329.67}$$

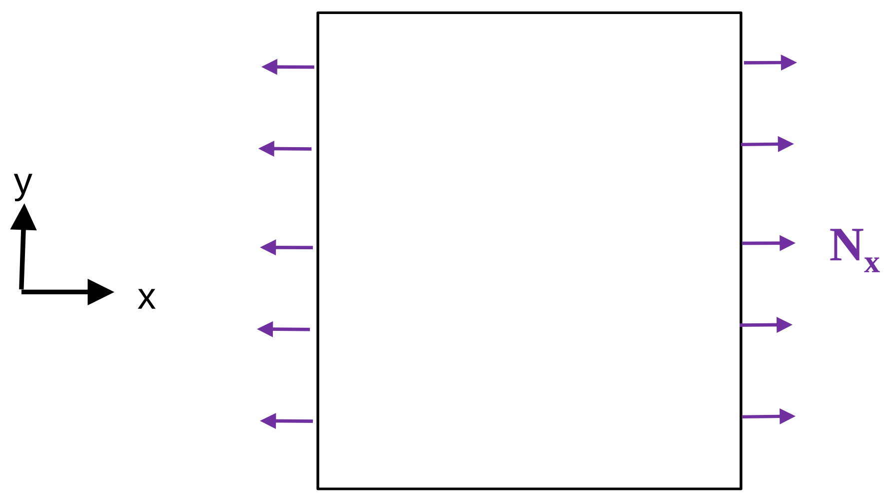
Free Body Diagram Output (Controlling Factored Loadcase)

Controlling Analysis Load: STRENGTH

	Nx,εx	Ny,εy	Nxy,γxy	Mx,κx	My,κy	Mxy,κxy	Qx	Qy
Virtual Loads	1098.9	329.67	0	0	0	0	0	0
Design-to Loads	1098.9	329.67	0	0	0	0	0	0
Design-to Deformation	0.001	0	0	0	0	0		

Ex 2 – Applied N_x, Free N_y





What will the strains look like?

Ex 2 – Applied N_x, Free N_y

Input (Per Load Case)

LIMIT-MECHANICAL Load Case #1 "one" (Mechanical Set #101, Thermal Set #201)

- Mechanical Load Set #101 "Load Set 101"
- Thermal Load Set #201 "Load Set 201"

Ref Temp | Temp | Pressure 0 | TT Grad

- FEA Loads - Projects Only
- User Loads

	Nx, εx	Ny, εy	Nxy, γxy	Mx, κx	My, κy	Mxy, κxy	Qx	Qy
Applied Unit Value	Load	Free	Constraine	Constraine	Constraine	Constraine	Load	Load
For Strength Analysis	1000							
For Buckling Analysis	1000							

$$\varepsilon_x = A_{11}^{-1} N_x + A_{12}^{-1} \cancel{N_y} \qquad A_{11}^{-1} = \frac{1}{Et}$$

$$= \frac{1}{(10)(.1)}(1000) \boxed{= 0.001}$$

$$\varepsilon_y = A_{21}^{-1} N_x + A_{22}^{-1} \cancel{N_y} \qquad A_{21}^{-1} = \frac{-\nu}{Et}$$

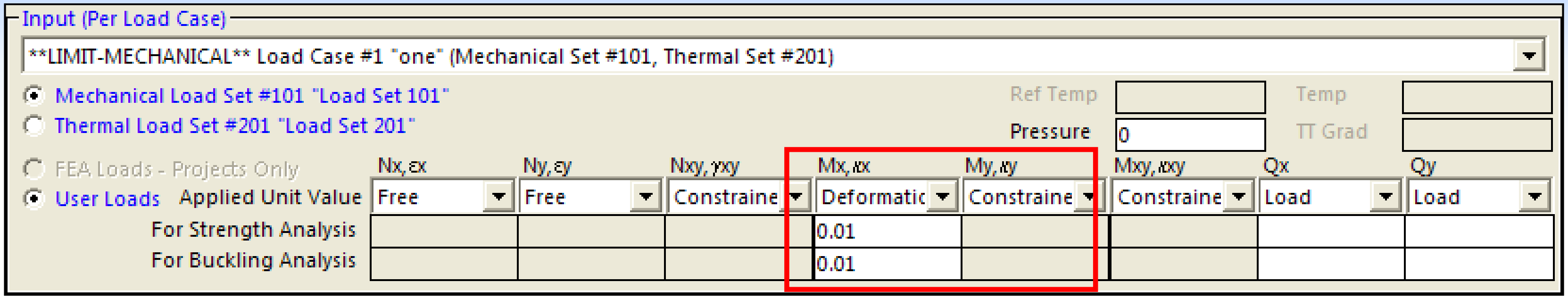
$$= \frac{-(0.3)}{(10)(.1)}(1000) \boxed{= -0.0003}$$

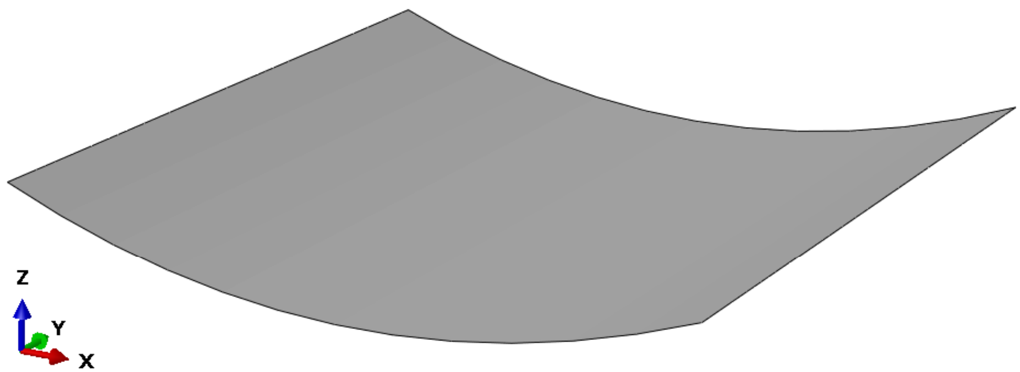
Free Body Diagram Output (Controlling Factored Loadcase)

Controlling Analysis Load: STRENGTH

	Nx, εx	Ny, εy	Nxy, γxy	Mx, κx	My, κy	Mxy, κxy	Qx	Qy
Virtual Loads								
Design-to Loads	1000	0	0	0	0	0	0	0
Design-to Deformation	0.001	-2.999999E-04	0	0	0	0		

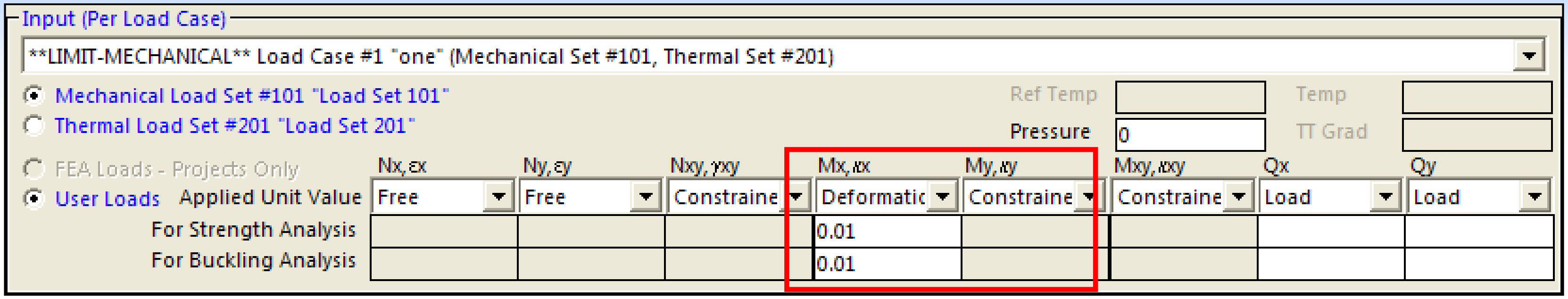
Ex 3 – Applied κ_x, Constrained κ_y

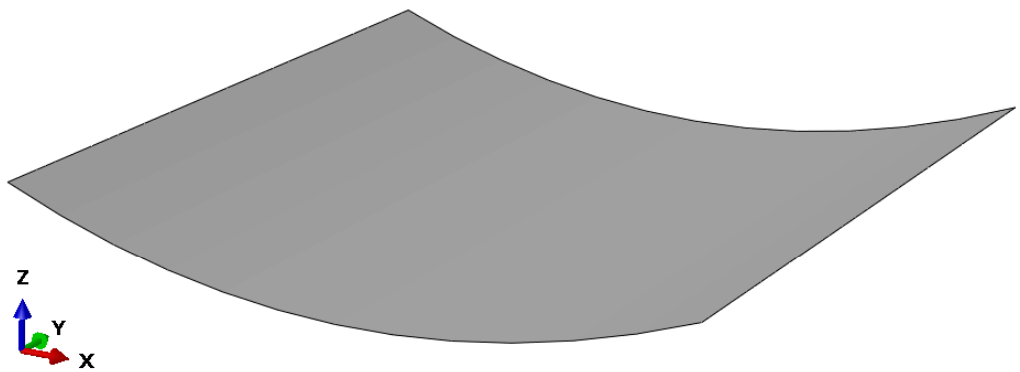




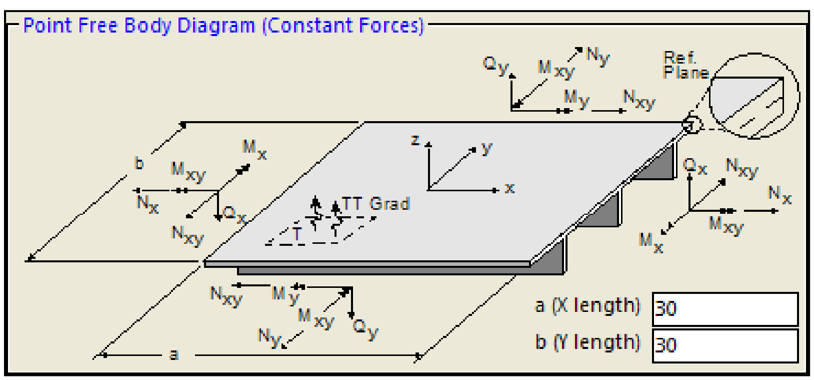
What will the loads look like?

Ex 3 – Applied κ_x, Constrained κ_y





What will the loads look like?



Ex 3 – Applied κ_x Constrained κ_y

Input (Per Load Case)

LIMIT-MECHANICAL Load Case #1 "one" (Mechanical Set #101, Thermal Set #201)

- (•) Mechanical Load Set #101 "Load Set 101"
- ( ) Thermal Load Set #201 "Load Set 201"

Ref Temp: | Temp: | Pressure: 0 | TT Grad:

- ( ) FEA Loads - Projects Only
- (•) User Loads

	Nx, εx	Ny, εy	Nxy, γxy	Mx, κx	My, κy	Mxy, κxy	Qx	Qy
Applied Unit Value	Free	Free	Constraine	Deformatic	Constraine	Constraine	Load	Load
For Strength Analysis				0.01				
For Buckling Analysis				0.01				

$$Mx = D_{11}\kappa_x + D_{12}\cancel{\kappa_y}$$

$$= (915.8)(0.01)$$

$$\boxed{= 9.158}$$

$$My = D_{21}\kappa_x = \nu D_{11}\kappa_x$$

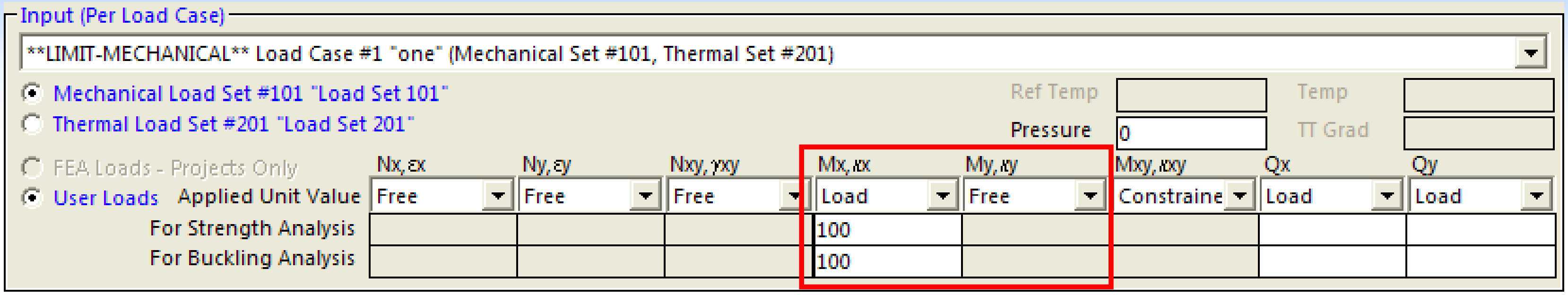
$$= (0.3)(915.8)(0.01)$$

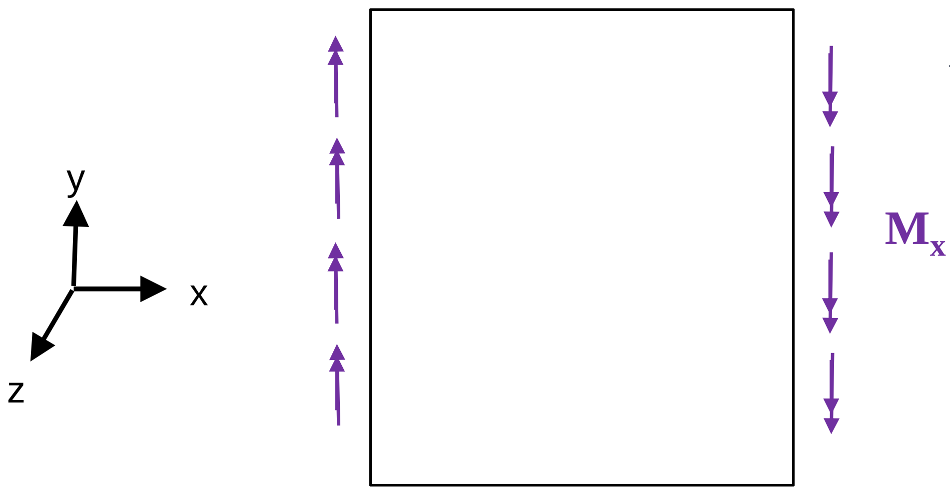
$$\boxed{= 2.747}$$

Free Body Diagram Output (Controlling Factored Loadcase)

Controlling Analysis Load: STRENGTH	Nx, εx	Ny, εy	Nxy, γxy	Mx, κx	My, κy	Mxy, κxy	Qx	Qy
Virtual Loads	0	0	0	9.15751	2.74725	0	0	0
Design-to Loads	0	0	0	9.15751	2.74725	0	0	0
Design-to Deformation	0	0	0	0.01	0	0		

Ex 4 – Applied M_x, Free M_y





What will the strains look like?

Ex 4 – Applied M_x, Free M_y

Input (Per Load Case)

LIMIT-MECHANICAL Load Case #1 "one" (Mechanical Set #101, Thermal Set #201)

- Mechanical Load Set #101 "Load Set 101"
- Thermal Load Set #201 "Load Set 201"
- FEA Loads - Projects Only
- User Loads

Ref Temp: | Temp: | Pressure: 0 | TT Grad:

	Nx, εx	Ny, εy	Nxy, γxy	Mx, κx	My, κy	Mxy, κxy	Qx	Qy
Applied Unit Value	Free	Free	Free	Load	Free	Constraine	Load	Load
For Strength Analysis				100				
For Buckling Analysis				100				

$$\kappa_x = D_{11}^{-1} M_x + D_{12}^{-1} \cancel{M}_y \qquad D_{11}^{-1} = \frac{12}{Et^3}$$

$$= \frac{12}{\left(10 \times 10^6\right)(.1)^3}(100) \boxed{= 0.12}$$

$$\kappa_y = D_{21}^{-1} M_x + D_{22}^{-1} \cancel{M}_y \qquad D_{21}^{-1} = \frac{-12\nu}{Et^2}$$

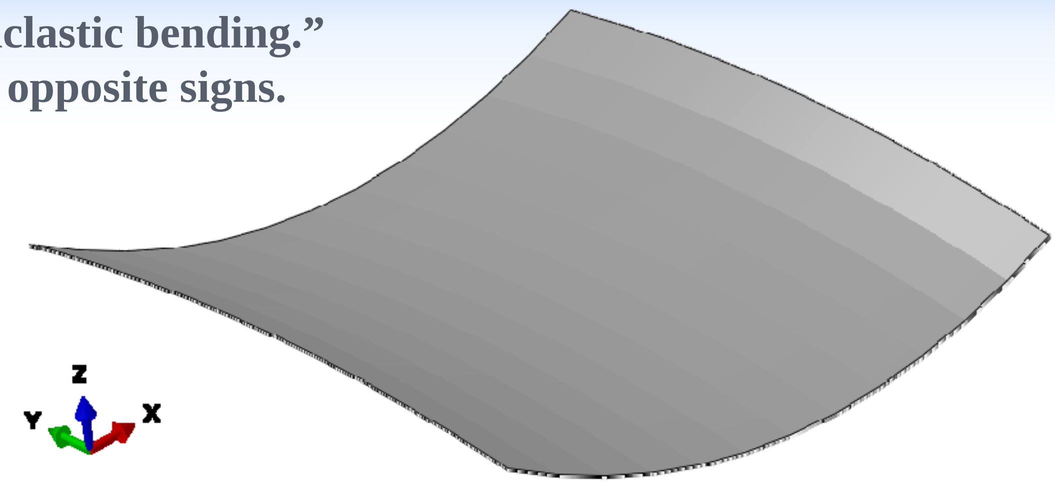
$$= \frac{-12(0.3)}{\left(10 \times 10^6\right)(.1)^3}(100) \boxed{= -0.036}$$

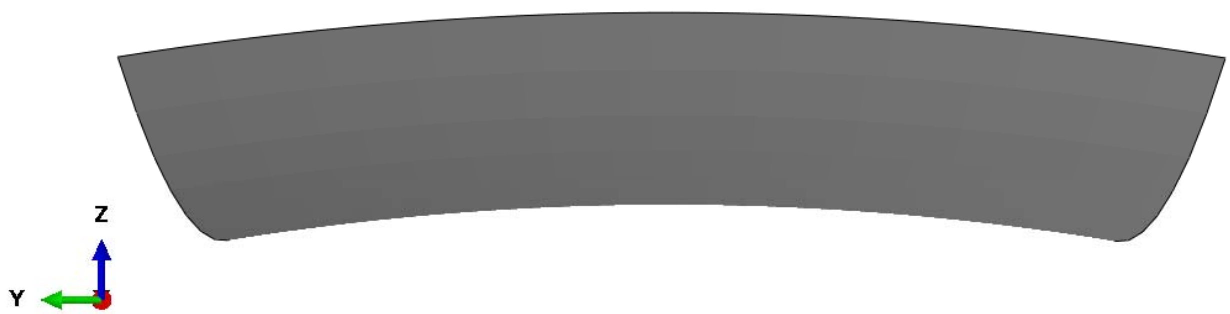
Free Body Diagram Output (Controlling Factored Loadcase)

Controlling Analysis Load: STRENGTH	Nx, εx	Ny, εy	Nxy, γxy	Mx, κx	My, κy	Mxy, κxy	Qx	Qy
Virtual Loads								
Design-to Loads	0	0	0	100	0	0	0	0
Design-to Deformation	0	0	0	0.12	-3.599999E-02	0		

Ex 4 – Applied M_x, Free M_y

Example of "anticlastic bending."
Curvatures have opposite signs.

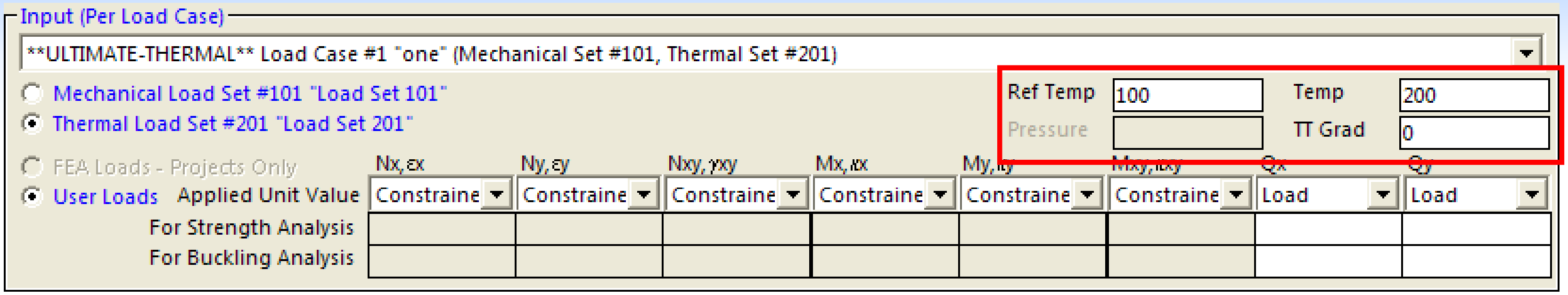


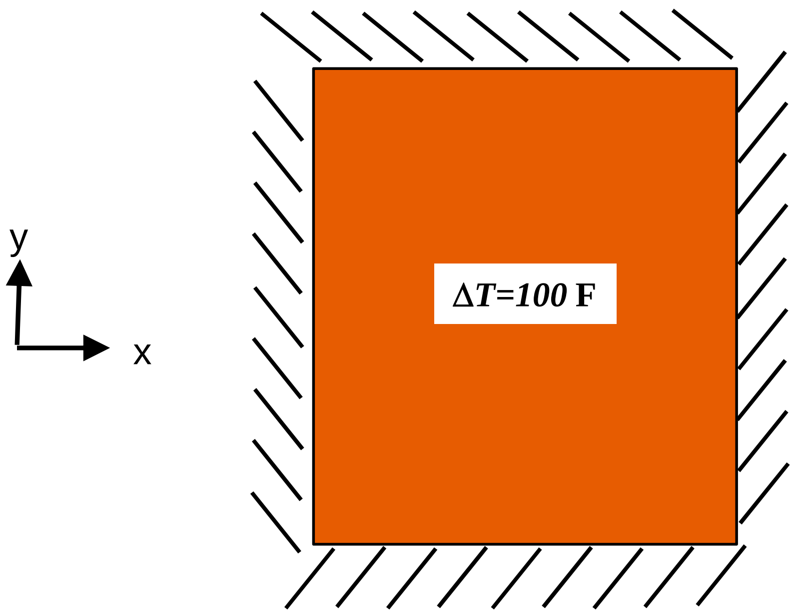


Free Body Diagram Output (Controlling Factored Loadcase)

Controlling Analysis Load: STRENGTH	Nx, εx	Ny, εy	Nxy, γxy	Mx, κx	My, κy	Mxy, κxy	Qx	Qy
Virtual Loads								
Design-to Loads	0	0	0	100	0	0	0	0
Design-to Deformation	0	0	0	0.12	-3.599999E-02	0		

Ex 5 – Thermal: Applied ΔT Constrained





What will the loads look like?

Positive, negative or zero?

Ex 5 –Thermal: Applied ΔT Constrained

Input (Per Load Case)

ULTIMATE-THERMAL Load Case #1 "one" (Mechanical Set #101, Thermal Set #201)

- Mechanical Load Set #101 "Load Set 101"
- Thermal Load Set #201 "Load Set 201"

Ref Temp: 100 | Temp: 200
Pressure: | TT Grad: 0

- FEA Loads - Projects Only
- User Loads

	Nx, εx	Ny, εy	Nxy, γxy	Mx, κx	My, κy	Mxy, κxy	Qx	Qy
Applied Unit Value	Constraine	Constraine	Constraine	Constraine	Constraine	Constraine	Load	Load
For Strength Analysis								
For Buckling Analysis								

Free Body Diagram Output (Controlling Factored Loadcase)

Controlling Analysis Load: BUCKLING

	Nx, εx	Ny, εy	Nxy, γxy	Mx, κx	My, κy	Mxy, κxy	Qx	Qy
Virtual Loads	-1714.29	-1714.29	0	0	0	0	0	0
Design-to Loads	-1714.29	-1714.29	0	0	0	0	0	0
Design-to Deformation	-0.0012	-0.0012	0	0	0	0		

Ex 5 –Thermal: Applied ΔT Constrained

Input (Per Load Case)

ULTIMATE-THERMAL Load Case #1 "one" (Mechanical Set #101, Thermal Set #201)

- Mechanical Load Set #101 "Load Set 101"
- (selected) Thermal Load Set #201 "Load Set 201"

Ref Temp	100	Temp	200
Pressure		TT Grad	0

- FEA Loads - Projects Only
- (selected) User Loads

	Nx, εx	Ny, εy	Nxy, γxy	Mx, κx	My, κy	Mxy, κxy	Qx	Qy
Applied Unit Value	Constraine	Constraine	Constraine	Constraine	Constraine	Constraine	Load	Load
For Strength Analysis								
For Buckling Analysis								

Thermal Strain

$$\varepsilon_x^T = \varepsilon_y^T = \alpha \Delta T$$

$$= \left(12 \times 10^{-6}\right)\left(100\right)$$

$$= 0.0012$$

Free Body Diagram Output (Controlling Factored Loadcase)

Controlling Analysis Load: BUCKLING	Nx, εx	Ny, εy	Nxy, γxy	Mx, κx	My, κy	Mxy, κxy	Qx	Qy
Virtual Loads	-1714.29	-1714.29	0	0	0	0	0	0
Design-to Loads	-1714.29	-1714.29	0	0	0	0	0	0
Design-to Deformation	-0.0012	-0.0012	0	0	0	0		

Ex 5 –Thermal: Applied ΔT Constrained

Input (Per Load Case)

ULTIMATE-THERMAL Load Case #1 "one" (Mechanical Set #101, Thermal Set #201)

- Mechanical Load Set #101 "Load Set 101"
- Thermal Load Set #201 "Load Set 201"

Ref Temp: 100 | Temp: 200
Pressure: | TT Grad: 0

- FEA Loads - Projects Only
- User Loads

	Nx, εx	Ny, εy	Nxy, γxy	Mx, κx	My, κy	Mxy, κxy	Qx	Qy
Applied Unit Value	Constraine	Constraine	Constraine	Constraine	Constraine	Constraine	Load	Load
For Strength Analysis								
For Buckling Analysis								

Thermal Strain

$$\varepsilon_x^T = \varepsilon_y^T = \alpha \Delta T$$
$$= \left(12 \times 10^{-6}\right)\left(100\right)$$
$$= 0.0012$$

Strain Actual (Computed Properties Tab)

Deformation		
Strain X		0
Strain Y		0

Design-To Strain

$$\varepsilon_x^{Design-To} = \cancel{\varepsilon_x^{Actual}} - \varepsilon_x^T$$
$$\boxed{= -0.0012}$$

Free Body Diagram Output (Controlling Factored Loadcase)

Controlling Analysis Load: BUCKLING

	Nx, εx	Ny, εy	Nxy, γxy	Mx, κx	My, κy	Mxy, κxy	Qx	Qy
Virtual Loads	-1714.29	-1714.29	0	0	0	0	0	0
Design-to Loads	-1714.29	-1714.29	0	0	0	0	0	0
Design-to Deformation	-0.0012	-0.0012	0	0	0	0		

Ex 5 –Thermal: Applied ΔT Constrained

Input (Per Load Case)

ULTIMATE-THERMAL Load Case #1 "one" (Mechanical Set #101, Thermal Set #201)

- Mechanical Load Set #101 "Load Set 101"
- Thermal Load Set #201 "Load Set 201"

Ref Temp	100	Temp	200
Pressure		TT Grad	0

- FEA Loads - Projects Only
- User Loads

	Nx, εx	Ny, εy	Nxy, γxy	Mx, κx	My, κy	Mxy, κxy	Qx	Qy
Applied Unit Value	Constraine	Constraine	Constraine	Constraine	Constraine	Constraine	Load	Load
For Strength Analysis								
For Buckling Analysis								

Thermal Strain

$$\varepsilon_x^T = \varepsilon_y^T = \alpha \Delta T$$

$$= \left(12\times10^{-6}\right)(100)$$

$$= 0.0012$$

Strain Actual (Computed Properties Tab)

Deformation		
Strain X		0
Strain Y		0

Design-To Strain

$$\varepsilon_x^{Design-To} = \cancel{\varepsilon_x^{Actual}} - \varepsilon_x^T$$

$$\boxed{= -0.0012}$$

Design-To Force

$$N_x^{Design-To} = A_{11}\varepsilon_x^{Design-To} + A_{12}\varepsilon_y^{Design-To}$$

$$= \left(1.0989\times10^{6}\right)(-0.0012) + (0.3)\left(1.0989\times10^{6}\right)(-0.0012)$$

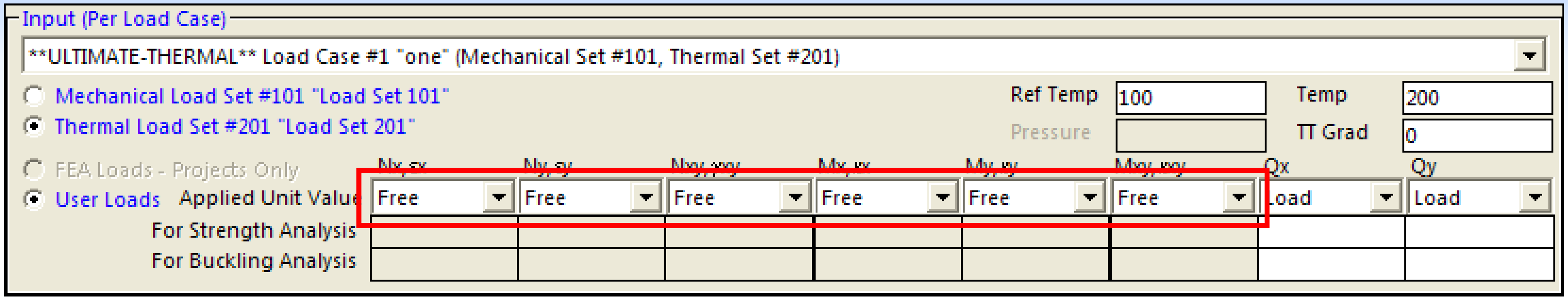
$$\boxed{= -1714.3}$$

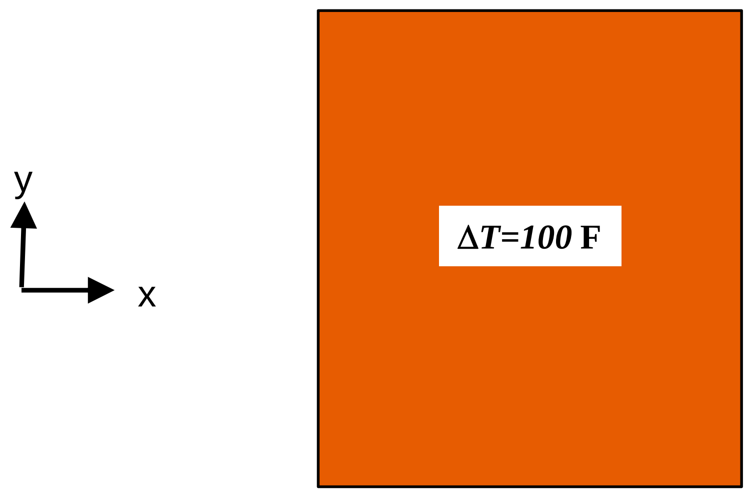
Free Body Diagram Output (Controlling Factored Loadcase)

Controlling Analysis Load: BUCKLING

	Nx, εx	Ny, εy	Nxy, γxy	Mx, κx	My, κy	Mxy, κxy	Qx	Qy
Virtual Loads	-1714.29	-1714.29	0	0	0	0	0	0
Design-to Loads	-1714.29	-1714.29	0	0	0	0	0	0
Design-to Deformation	-0.0012	-0.0012	0	0	0	0		

Ex 6 – Thermal: Applied ΔT, Edges Free





What will the loads look like?

Positive, negative or zero?

Ex 6 –Thermal: Applied ∆T Free

Input (Per Load Case)

ULTIMATE-THERMAL Load Case #1 "one" (Mechanical Set #101, Thermal Set #201)

- Mechanical Load Set #101 "Load Set 101"
- Thermal Load Set #201 "Load Set 201"

Ref Temp: 100 | Temp: 200
Pressure: | TT Grad: 0

- FEA Loads - Projects Only
- User Loads

	Nx,εx	Ny,εy	Nxy,γxy	Mx,κx	My,κy	Mxy,κxy	Qx	Qy
Applied Unit Value	Free	Free	Free	Free	Free	Free	Load	Load
For Strength Analysis								
For Buckling Analysis								

Free Body Diagram Output (Controlling Factored Loadcase)

Controlling Analysis Load: BUCKLING	Nx,εx	Ny,εy	Nxy,γxy	Mx,κx	My,κy	Mxy,κxy	Qx	Qy
Virtual Loads								
Design-to Loads								
Design-to Deformation	0	0	0	0	0	0		

HyperSizer®

Ex 6 –Thermal: Applied ΔT Free

Input (Per Load Case)

ULTIMATE-THERMAL Load Case #1 "one" (Mechanical Set #101, Thermal Set #201)

- Mechanical Load Set #101 "Load Set 101"
- Thermal Load Set #201 "Load Set 201"

Ref Temp: 100 | Temp: 200
Pressure: | TT Grad: 0

- FEA Loads - Projects Only
- User Loads

	Nx,εx	Ny,εy	Nxy,γxy	Mx,κx	My,κy	Mxy,κxy	Qx	Qy
Applied Unit Value	Free	Free	Free	Free	Free	Free	Load	Load
For Strength Analysis								
For Buckling Analysis								

Thermal Strain

$$\varepsilon_x^T = \varepsilon_y^T = \alpha \Delta T$$

$$= \left(12 \times 10^{-6}\right)\left(100\right)$$

$$= 0.0012$$

Free Body Diagram Output (Controlling Factored Loadcase)

Controlling Analysis Load: BUCKLING	Nx,εx	Ny,εy	Nxy,γxy	Mx,κx	My,κy	Mxy,κxy	Qx	Qy
Virtual Loads								
Design-to Loads								
Design-to Deformation	0	0	0	0	0	0		

Ex 6 –Thermal: Applied ΔT Free

Input (Per Load Case)

ULTIMATE-THERMAL Load Case #1 "one" (Mechanical Set #101, Thermal Set #201)

- ( ) Mechanical Load Set #101 "Load Set 101"
- (•) Thermal Load Set #201 "Load Set 201"

Ref Temp	100	Temp	200
Pressure		TT Grad	0

- ( ) FEA Loads - Projects Only
- (•) User Loads

	Nx, εx	Ny, εy	Nxy, γxy	Mx, κx	My, κy	Mxy, κxy	Qx	Qy
Applied Unit Value	Free	Free	Free	Free	Free	Free	Load	Load
For Strength Analysis								
For Buckling Analysis								

Thermal Strain

$$\varepsilon_x^T = \varepsilon_y^T = \alpha \Delta T$$

$$= \left(12 \times 10^{-6}\right)(100)$$

$$= 0.0012$$

Strain Actual (Computed Properties Tab)

	Deformation	
Strain X		0.0012
Strain Y		0.0012

Free Body Diagram Output (Controlling Factored Loadcase)

Controlling Analysis Load: BUCKLING	Nx, εx	Ny, εy	Nxy, γxy	Mx, κx	My, κy	Mxy, κxy	Qx	Qy
Virtual Loads								
Design-to Loads								
Design-to Deformation	0	0	0	0	0	0		

HyperSizer®

Ex 6 –Thermal: Applied ΔT Free

Input (Per Load Case)

ULTIMATE-THERMAL Load Case #1 "one" (Mechanical Set #101, Thermal Set #201)

- Mechanical Load Set #101 "Load Set 101"
- Thermal Load Set #201 "Load Set 201"

Ref Temp: 100 | Temp: 200
Pressure: | TT Grad: 0

- FEA Loads - Projects Only
- User Loads

	Nx,εx	Ny,εy	Nxy,γxy	Mx,κx	My,κy	Mxy,κxy	Qx	Qy
Applied Unit Value	Free	Free	Free	Free	Free	Free	Load	Load
For Strength Analysis								
For Buckling Analysis								

Thermal Strain

$$\varepsilon_x^T = \varepsilon_y^T = \alpha \Delta T$$

$$= \left(12 \times 10^{-6}\right)\left(100\right)$$

$$= 0.0012$$

Strain Actual (Computed Properties Tab)

Deformation		
Strain X		0.0012
Strain Y		0.0012

Design-To Strain

$$\varepsilon_x^{Actual} = \varepsilon_x^T$$

$$\varepsilon_x^{Design-To} = \varepsilon_x^{Actual} - \varepsilon_x^T$$

$$\boxed{= 0.0}$$

Free Body Diagram Output (Controlling Factored Loadcase)

Controlling Analysis Load: BUCKLING	Nx,εx	Ny,εy	Nxy,γxy	Mx,κx	My,κy	Mxy,κxy	Qx	Qy
Virtual Loads								
Design-to Loads								
Design-to Deformation	0	0	0	0	0	0		

Ex 6 –Thermal: Applied ΔT Free

Input (Per Load Case)

ULTIMATE-THERMAL Load Case #1 "one" (Mechanical Set #101, Thermal Set #201)

- Mechanical Load Set #101 "Load Set 101"
- Thermal Load Set #201 "Load Set 201" (selected)

Ref Temp	100	Temp	200
Pressure		TT Grad	0

- FEA Loads - Projects Only
- User Loads (selected)

	Nx,εx	Ny,εy	Nxy,γxy	Mx,κx	My,κy	Mxy,κxy	Qx	Qy
Applied Unit Value	Free	Free	Free	Free	Free	Free	Load	Load
For Strength Analysis								
For Buckling Analysis								

Thermal Strain

$$\varepsilon_x^T = \varepsilon_y^T = \alpha\Delta T$$
$$= \left(12\times10^{-6}\right)\left(100\right)$$
$$= 0.0012$$

Strain Actual (Computed Properties Tab)

	Deformation	
Strain X		0.0012
Strain Y		0.0012

Design-To Strain

$$\varepsilon_x^{Actual} = \varepsilon_x^T$$
$$\varepsilon_x^{Design-To} = \varepsilon_x^{Actual} - \varepsilon_x^T$$
$$\boxed{= 0.0}$$

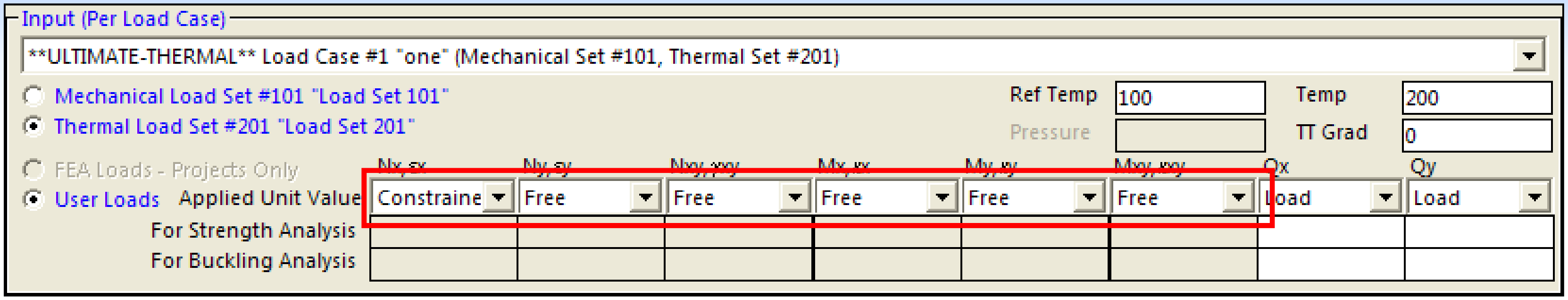
Design-To Force

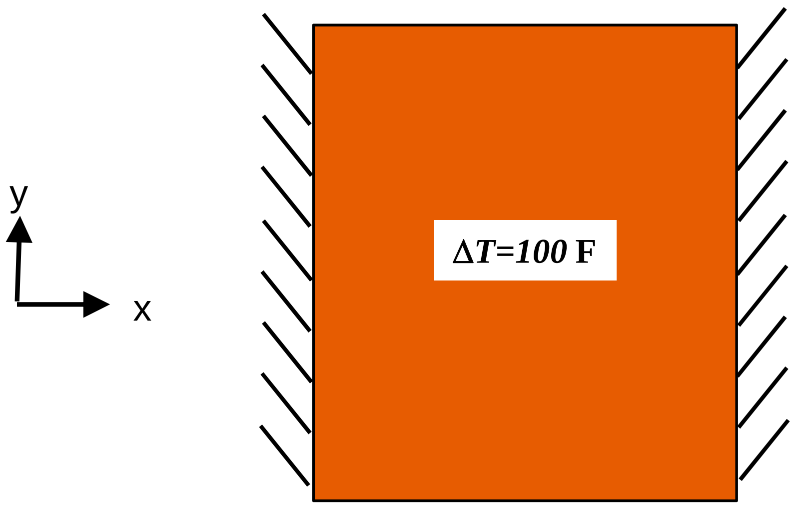
$$N_x^{Design-To} = A_{11}\cancel{\varepsilon_x^{Design-To}} + A_{12}\cancel{\varepsilon_y^{Design-To}}$$
$$\boxed{= 0.0}$$

Free Body Diagram Output (Controlling Factored Loadcase)

Controlling Analysis Load: BUCKLING	Nx,εx	Ny,εy	Nxy,γxy	Mx,κx	My,κy	Mxy,κxy	Qx	Qy
Virtual Loads								
Design-to Loads								
Design-to Deformation	0	0	0	0	0	0		

Ex 7 – Applied ∆T, Partially Constrained





What will the loads look like?

Positive, negative or zero?

Ex 7 –Applied ΔT Partially Constrained

Input (Per Load Case)

ULTIMATE-THERMAL Load Case #1 "one" (Mechanical Set #101, Thermal Set #201)

- ○ Mechanical Load Set #101 "Load Set 101"
- ◉ Thermal Load Set #201 "Load Set 201"

Ref Temp	100	Temp	200
Pressure		TT Grad	0

- ○ FEA Loads - Projects Only
- ◉ User Loads

	Nx, εx	Ny, εy	Nxy, γxy	Mx, κx	My, κy	Mxy, κxy	Qx	Qy
Applied Unit Value	Constraine	Free	Free	Free	Free	Free	Load	Load
For Strength Analysis								
For Buckling Analysis								

Free Body Diagram Output (Controlling Factored Loadcase)

Controlling Analysis Load: BUCKLING	Nx, εx	Ny, εy	Nxy, γxy	Mx, κx	My, κy	Mxy, κxy	Qx	Qy
Virtual Loads	-1200	0	0	0	0	0	0	0
Design-to Loads	-1200	0	0	0	0	0	0	0
Design-to Deformation	-0.0012	3.599999E-04	0	0	0	0		

Ex 7 –Applied ΔT Partially Constrained

Input (Per Load Case)

ULTIMATE-THERMAL Load Case #1 "one" (Mechanical Set #101, Thermal Set #201)

- Mechanical Load Set #101 "Load Set 101"
- Thermal Load Set #201 "Load Set 201"

Ref Temp	100	Temp	200
Pressure		TT Grad	0

- FEA Loads - Projects Only
- User Loads

	Nx, εx	Ny, εy	Nxy, γxy	Mx, κx	My, κy	Mxy, κxy	Qx	Qy
Applied Unit Value	Constraine	Free	Free	Free	Free	Free	Load	Load
For Strength Analysis								
For Buckling Analysis								

Thermal Strain

$$\varepsilon_x^T = \varepsilon_y^T = \alpha \Delta T$$

$$= \left(12 \times 10^{-6}\right)(100)$$

$$= 0.0012$$

Free Body Diagram Output (Controlling Factored Loadcase)

Controlling Analysis Load: BUCKLING	Nx, εx	Ny, εy	Nxy, γxy	Mx, κx	My, κy	Mxy, κxy	Qx	Qy
Virtual Loads	-1200	0	0	0	0	0	0	0
Design-to Loads	-1200	0	0	0	0	0	0	0
Design-to Deformation	-0.0012	3.599999E-04	0	0	0	0		

Ex 7 –Applied ΔT Partially Constrained

Input (Per Load Case)

ULTIMATE-THERMAL Load Case #1 "one" (Mechanical Set #101, Thermal Set #201)

- Mechanical Load Set #101 "Load Set 101"
- Thermal Load Set #201 "Load Set 201" (selected)

Ref Temp	100	Temp	200
Pressure		TT Grad	0

- FEA Loads - Projects Only
- User Loads (selected)

	Nx, εx	Ny, εy	Nxy, γxy	Mx, κx	My, κy	Mxy, κxy	Qx	Qy
Applied Unit Value	Constraine	Free	Free	Free	Free	Free	Load	Load
For Strength Analysis								
For Buckling Analysis								

Thermal Strain

$$\varepsilon_x^T = \varepsilon_y^T = \alpha \Delta T$$

$$= \left(12 \times 10^{-6}\right)\left(100\right)$$

$$= 0.0012$$

Strain Actual (Computed Properties Tab)

Deformation		
Strain X		0
Strain Y		0.00156

$$\varepsilon_y^{Actual} = \varepsilon_y^T + \nu \varepsilon_x^T$$

$$\boxed{= 0.00156}$$

Free Body Diagram Output (Controlling Factored Loadcase)

Controlling Analysis Load: BUCKLING	Nx, εx	Ny, εy	Nxy, γxy	Mx, κx	My, κy	Mxy, κxy	Qx	Qy
Virtual Loads	-1200	0	0	0	0	0	0	0
Design-to Loads	-1200	0	0	0	0	0	0	0
Design-to Deformation	-0.0012	3.599999E-04	0	0	0	0		

Ex 7 –Applied ΔT Partially Constrained

Input (Per Load Case)

ULTIMATE-THERMAL Load Case #1 "one" (Mechanical Set #101, Thermal Set #201)

- Mechanical Load Set #101 "Load Set 101"
- Thermal Load Set #201 "Load Set 201"

Ref Temp	100	Temp	200
Pressure		TT Grad	0

- FEA Loads - Projects Only
- User Loads

	Nx, εx	Ny, εy	Nxy, γxy	Mx, κx	My, κy	Mxy, κxy	Qx	Qy
Applied Unit Value	Constraine	Free	Free	Free	Free	Free	Load	Load
For Strength Analysis								
For Buckling Analysis								

Thermal Strain

$$\varepsilon_x^T = \varepsilon_y^T = \alpha \Delta T$$

$$= \left(12 \times 10^{-6}\right)\left(100\right)$$

$$= 0.0012$$

Strain Actual (Computed Properties Tab)

	Deformation	
Strain X		0
Strain Y		0.00156

$$\varepsilon_y^{Actual} = \varepsilon_y^T + \nu \varepsilon_x^T$$

$$\boxed{= 0.00156}$$

Design-To Strain

$$\varepsilon_x^{Actual} = \varepsilon_x^T$$

$$\varepsilon_x^{Design-To} = \cancel{\varepsilon_x^{Actual}} - \varepsilon_x^T$$

$$\boxed{= -0.0012}$$

$$\varepsilon_y^{Design-To} = \varepsilon_y^{Actual} - \varepsilon_y^T$$

$$\varepsilon_y^{Design-To} = \left(0.00156\right) - \left(0.0012\right)$$

$$\boxed{= 0.00036}$$

Free Body Diagram Output (Controlling Factored Loadcase)

Controlling Analysis Load: BUCKLING	Nx, εx	Ny, εy	Nxy, γxy	Mx, κx	My, κy	Mxy, κxy	Qx	Qy
Virtual Loads	-1200	0	0	0	0	0	0	0
Design-to Loads	-1200	0	0	0	0	0	0	0
Design-to Deformation	-0.0012	3.599999E-04	0	0	0	0		

Ex 7 –Applied ΔT Partially Constrained

Input (Per Load Case)

ULTIMATE-THERMAL Load Case #1 "one" (Mechanical Set #101, Thermal Set #201)

Mechanical Load Set #101 "Load Set 101"

Thermal Load Set #201 "Load Set 201"

Ref Temp: 100 | Temp: 200

Pressure: | TT Grad: 0

FEA Loads - Projects Only

User Loads

	Nx, εx	Ny, εy	Nxy, γxy	Mx, κx	My, κy	Mxy, κxy	Qx	Qy
Applied Unit Value	Constraine	Free	Free	Free	Free	Free	Load	Load
For Strength Analysis								
For Buckling Analysis								

Thermal Strain

$$\varepsilon_x^T = \varepsilon_y^T = \alpha \Delta T$$

$$= \left(12 \times 10^{-6}\right)\left(100\right)$$

$$= 0.0012$$

Strain Actual (Computed Properties Tab)

Deformation		
Strain X		0
Strain Y		0.00156

$$\varepsilon_y^{Actual} = \varepsilon_y^T + \nu \varepsilon_x^T$$

$$\boxed{= 0.00156}$$

Design-To Strain

$$\varepsilon_x^{Actual} = \varepsilon_x^T$$

$$\varepsilon_x^{Design-To} = \cancel{\varepsilon_x^{Actual}} - \varepsilon_x^T$$

$$\boxed{= -0.0012}$$

$$\varepsilon_y^{Design-To} = \varepsilon_y^{Actual} - \varepsilon_y^T$$

$$\varepsilon_y^{Design-To} = \left(0.00156\right) - \left(0.0012\right)$$

$$\boxed{= 0.00036}$$

Design-To Force

$$N_x^{Design-To} = A_{11}\varepsilon_x^{Design-To} + A_{12}\varepsilon_y^{Design-To}$$

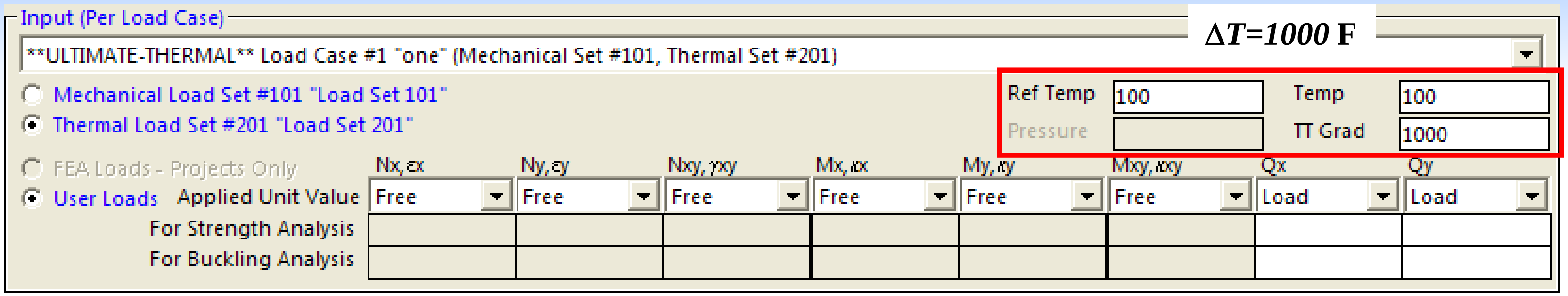
$$= \left(1.0989 \times 10^6\right)\left(-0.0012\right) + \left(0.3\right)\left(1.0989 \times 10^6\right)\left(.00036\right)$$

$$\boxed{= -1200}$$

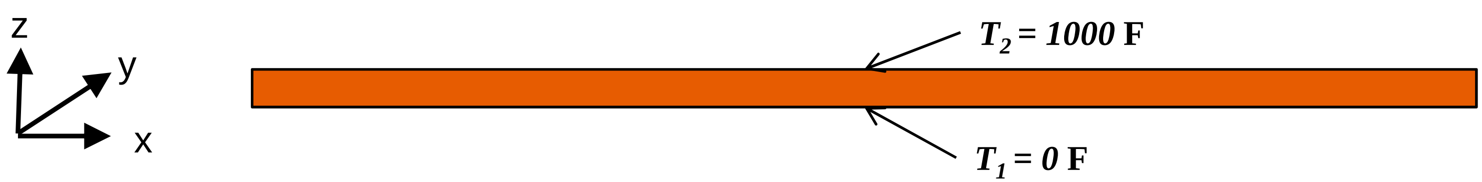
Free Body Diagram Output (Controlling Factored Loadcase)

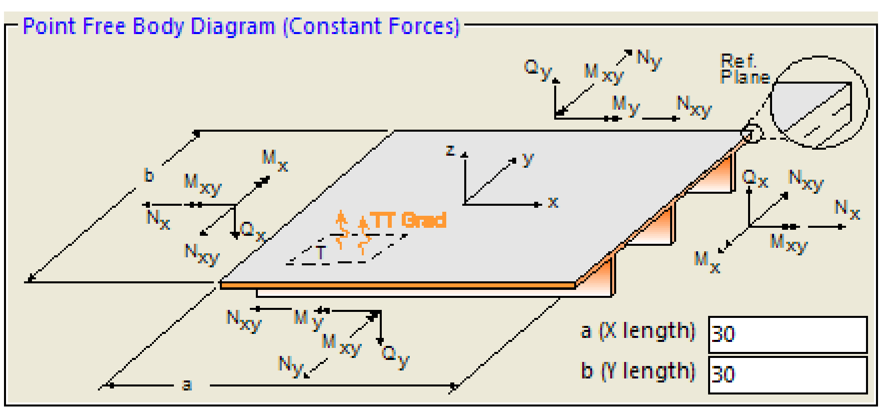
Controlling Analysis Load: BUCKLING	Nx, εx	Ny, εy	Nxy, γxy	Mx, κx	My, κy	Mxy, κxy	Qx	Qy
Virtual Loads	-1200	0	0	0	0	0	0	0
Design-to Loads	-1200	0	0	0	0	0	0	0
Design-to Deformation	-0.0012	3.599999E-04	0	0	0	0		

Ex 8 – Thru-Thickness ΔT, Edges Free



What will the strains look like?





Ex 8 – Thru-Thickness ΔT, Edges Free

Input (Per Load Case)

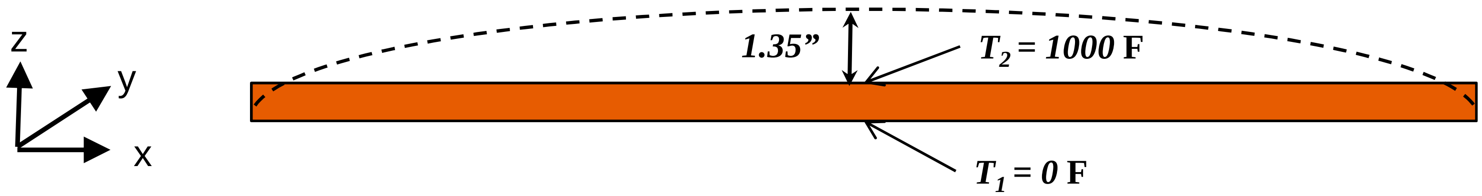
ULTIMATE-THERMAL Load Case #1 "one" (Mechanical Set #101, Thermal Set #201)

- Mechanical Load Set #101 "Load Set 101"
- Thermal Load Set #201 "Load Set 201"

Ref Temp: 100 | Temp: 100
Pressure: | TT Grad: 1000

- FEA Loads - Projects Only
- User Loads

	Nx, εx	Ny, εy	Nxy, γxy	Mx, κx	My, κy	Mxy, κxy	Qx	Qy
Applied Unit Value	Free	Free	Free	Free	Free	Free	Load	Load
For Strength Analysis								
For Buckling Analysis								



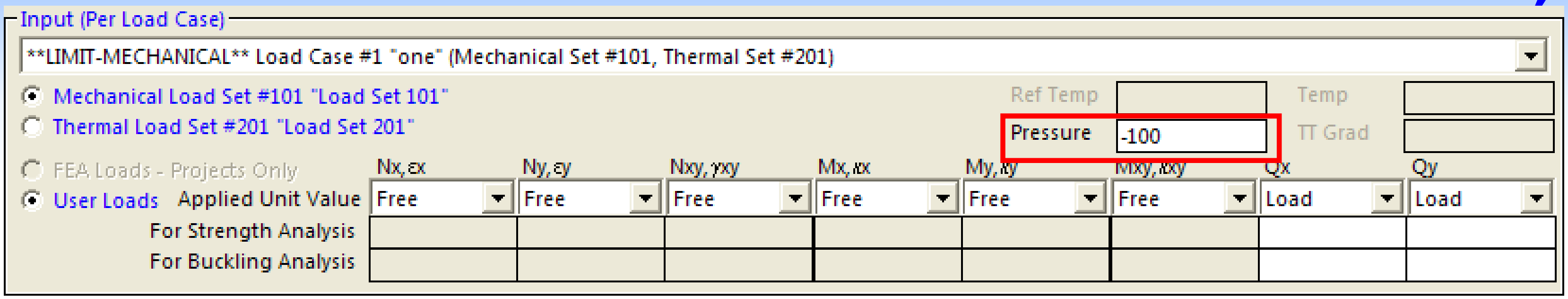
Deformation

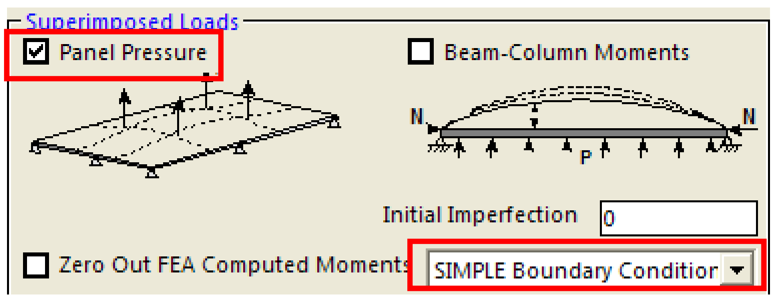
Strain X		-0.0006
Strain Y		-0.0006
Curvature X		-0.012
Curvature Y		-0.012
Midspan Deflection		1.35

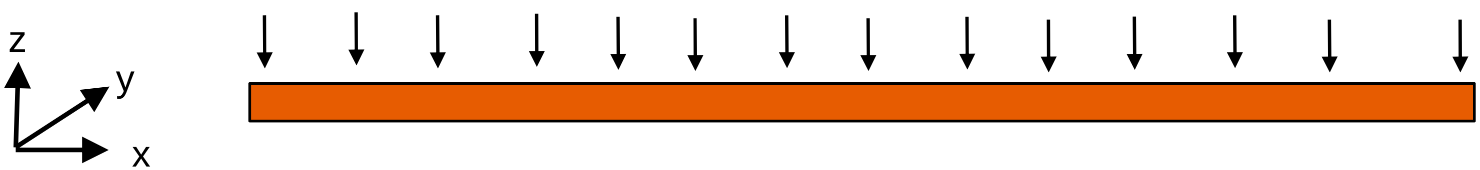
Free Body Diagram Output (Controlling Factored Loadcase)

Controlling Analysis Load: BUCKLING	Nx, εx	Ny, εy	Nxy, γxy	Mx, κx	My, κy	Mxy, κxy	Qx	Qy
Virtual Loads								
Design-to Loads								
Design-to Deformation	0	0	0	0	0	0		

Ex 9 – Panel Pressure







Ex 9 – Panel Pressure

Input (Per Load Case)

LIMIT-MECHANICAL Load Case #1 "one" (Mechanical Set #101, Thermal Set #201)

- (•) Mechanical Load Set #101 "Load Set 101"
- ( ) Thermal Load Set #201 "Load Set 201"

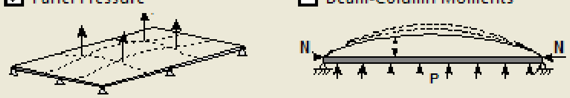
Ref Temp		Temp	
Pressure	-100	TT Grad	

- ( ) FEA Loads - Projects Only
- (•) User Loads

	Nx, εx	Ny, εy	Nxy, γxy	Mx, κx	My, κy	Mxy, κxy	Qx	Qy
Applied Unit Value	Free	Free	Free	Free	Free	Free	Load	Load
For Strength Analysis								
For Buckling Analysis								

Superimposed Loads

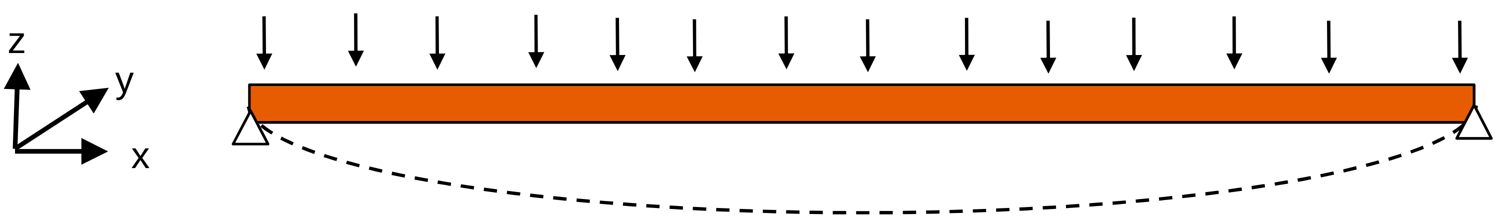
[x] Panel Pressure [ ] Beam-Column Moments



Initial Imperfection: 0

[ ] Zero Out FEA Computed Moments — SIMPLE Boundary Condition

	Mx	My	Qx	Qy
MidSpan	4309.657	4309.657	0	0
EdgeCntr	0	0	-997.5873	-997.5873

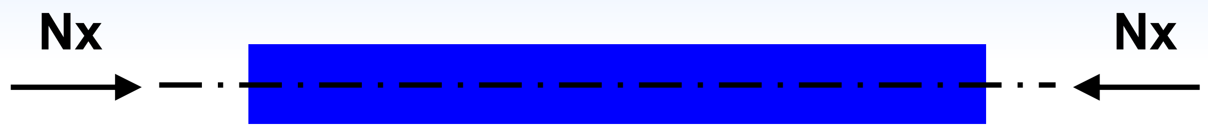


Free Body Diagram Output (Controlling Factored Loadcase)

Controlling Analysis Load: STRENGTH

	Nx, εx	Ny, εy	Nxy, γxy	Mx, κx	My, κy	Mxy, κxy	Qx	Qy
Virtual Loads								
Design-to Loads	0	0	0	4309.66	4309.66	0	0	0
Design-to Deformation	0	0	0	3.620112	3.620112	0		

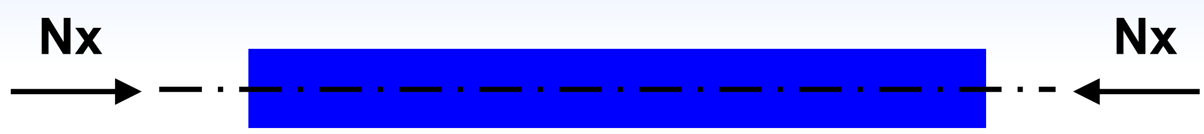
Panel Edge Loading



HyperSizer®

Panel Edge Loading

Single Isotropic Sheet



B Matrix = 0
No Membrane-Bending Coupling

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} A & 0 \\ 0 & D \end{bmatrix} \begin{Bmatrix} \varepsilon \\ \kappa \end{Bmatrix}$$

Panel Edge Loading

Single Isotropic Sheet

B Matrix = 0
No Membrane-Bending Coupling

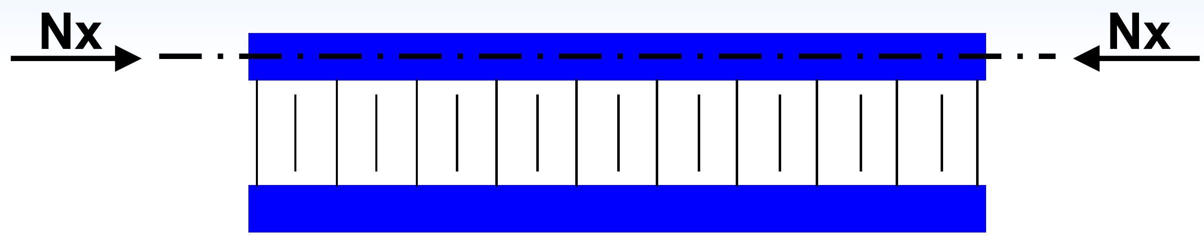
$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} A & 0 \\ 0 & D \end{bmatrix} \begin{Bmatrix} \varepsilon \\ \kappa \end{Bmatrix}$$

$$\boxed{N = A\varepsilon}$$

$$\boxed{M = D\kappa}$$

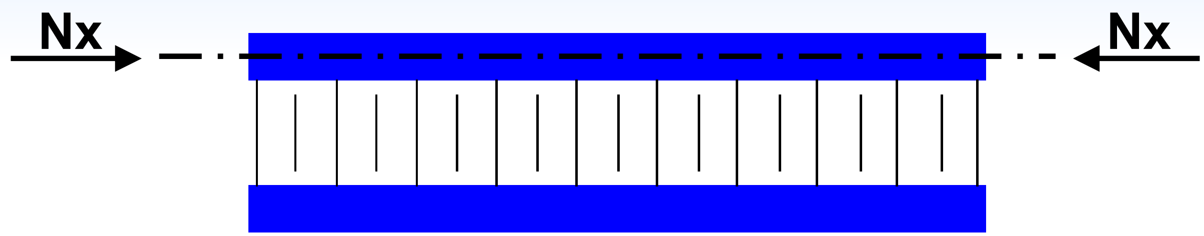
Panel Edge Loading

Symmetric Honeycomb Sandwich (Note Reference Plane)



Panel Edge Loading

Symmetric Honeycomb Sandwich
(Note Reference Plane)

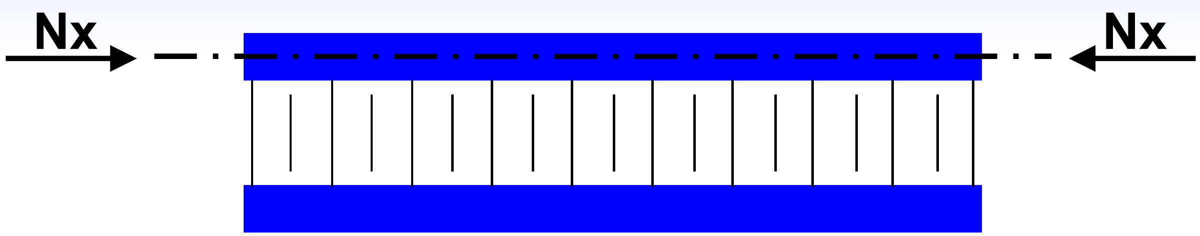


$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} \varepsilon \\ \kappa \end{Bmatrix}$$

B Matrix ≠ 0
Membrane-bending
Coupling is present

Panel Edge Loading

Symmetric Honeycomb Sandwich
(Note Reference Plane)



B Matrix $\neq 0$
Membrane-bending
Coupling is present

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} \varepsilon \\ \kappa \end{Bmatrix}$$

$$\boxed{N = A\varepsilon + B\kappa}$$

$$\boxed{M = B\varepsilon + D\kappa}$$

Free Body Diagram Math

Calculated by HyperSizer

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$$

Free Body Diagram Math

Calculated by HyperSizer

"Knowns"

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$$

Free Body Diagram Math

"Unknowns" — Calculated by HyperSizer — "Knowns"

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$$

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Free Body Diagram Math

"Unknowns" | Calculated by HyperSizer | "Knowns"

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$$

Unknowns on left, Knowns on right

Free Body Diagram Math – FEM Import

FORCES

$$\left\{ \begin{matrix} \\ \\ \\ \\ \\ \\ \end{matrix} \right\} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \left\{ \begin{matrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{matrix} \right\}$$

Free Body Diagram Math – FEM Import

STRAINS **FORCES**

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix}$$

Free Body Diagram Math – FEM Import

$$\underbrace{\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}}_{\text{STRAINS}} = \underbrace{\begin{bmatrix} A & B \\ B & D \end{bmatrix}^{-1}}_{\text{Inverted Matrix}} \underbrace{\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix}}_{\text{FORCES}}$$

Free Body Diagram Math – FEM Import

$$
\underbrace{\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}}_{\text{STRAINS}} = \underbrace{\begin{bmatrix} A & B \\ B & D \end{bmatrix}^{-1}}_{\text{Inverted Matrix}} \underbrace{\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix}}_{\text{FORCES}}
$$

$$
\varepsilon_x = A^{-1}_{11} N_x + A^{-1}_{12} N_Y + \ldots
$$

Free Body Diagram Math – FEM Import

$$\underbrace{\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}}_{\text{STRAINS}} = \underbrace{\begin{bmatrix} A & B \\ B & D \end{bmatrix}^{-1}_{6x6}}_{\text{Inverted Matrix}} \underbrace{\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix}}_{\text{FORCES}}$$

$$\varepsilon_x = A^{-1}_{11} N_x + A^{-1}_{12} N_Y + \ldots$$

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Free Body Diagram Math – FEM Import

$$\underbrace{\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}}_{\text{STRAINS}} = \underbrace{\begin{bmatrix} A & B \\ B & D \end{bmatrix}^{-1}_{6x6}}_{\text{Inverted Matrix}} \underbrace{\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix}}_{\text{FORCES}}$$

HyperSizer®

Free Body Diagram Math – FEM Import

$$
\underbrace{\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}}_{\text{STRAINS}} = \underbrace{\begin{bmatrix} A & B \\ B & D \end{bmatrix}^{-1}_{6\text{x}6}}_{\text{Inverted Matrix}} \underbrace{\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix}}_{\text{FORCES}}
$$

When coupling HyperSizer with a FEM, the FEA computed forces are imported to compute panel strains and curvatures this way. (At the reference plane)

FBD Math – Workspace Loads

Specifed Strain

	N_x, ε_x	N_y, ε_y	N_{xy}, γ_{xy}	M_x, κ_x	M_y, κ_y	M_{xy}, κ_{xy}
Applied Unit Value	Deformation	Constrained	Constrained	Constrained	Constrained	Constrained
For Strength Analysis	0.01					
For Buckling Analysis	0.01					

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$$

FBD Math – Workspace Loads

Specifed Strain

	Nx, εx	Ny, εy	Nxy, γxy	Mx, κx	My, κy	Mxy, κxy
Applied Unit Value	Deformation	Constrained	Constrained	Constrained	Constrained	Constrained
For Strength Analysis	0.01					
For Buckling Analysis	0.01					

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} 0.01 \\ \varepsilon_y \\ \varepsilon_{xy} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$$

HyperSizer®

FBD Math – Workspace Loads

Specifed Strain

	Nx, εx	Ny, εy	Nxy, γxy	Mx, κx	My, κy	Mxy, κxy
Applied Unit Value	Deformation	Constrained	Constrained	Constrained	Constrained	Constrained
For Strength Analysis	0.01					
For Buckling Analysis	0.01					

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} 0.01 \\ 0.0 \\ 0.0 \\ 0.0 \\ 0.0 \\ 0.0 \end{Bmatrix}$$

FBD Math – Workspace Loads

Specifed Strain

	N_x, ε_x	N_y, ε_y	N_{xy}, γ_{xy}	M_x, κ_x	M_y, κ_y	M_{xy}, κ_{xy}
Applied Unit Value	Deformation	Constrained	Constrained	Constrained	Constrained	Constrained
For Strength Analysis	0.01					
For Buckling Analysis	0.01					

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} 0.01 \\ 0.0 \\ 0.0 \\ 0.0 \\ 0.0 \\ 0.0 \end{Bmatrix}$$

$$N_x = A_{11}\,\varepsilon_x + A_{12} \quad + 0.0$$

$$N_y = A_{21}\,\varepsilon_x + A_{22} \quad + 0.0$$

FBD Math – Workspace Loads

Specifed Load

	Nx, εx	Ny, εy	Nxy, γxy	Mx, κx	My, κy	Mxy, κxy
Applied Unit Value	Load	Constrained	Constrained	Constrained	Constrained	Constrained
For Strength Analysis	-100					
For Buckling Analysis	-100					

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$$

FBD Math – Workspace Loads

Specifed Load

	N_x, ε_x	N_y, ε_y	N_{xy}, γ_{xy}	M_x, κ_x	M_y, κ_y	M_{xy}, κ_{xy}
Applied Unit Value	Load	Constrained	Constrained	Constrained	Constrained	Constrained
For Strength Analysis	-100					
For Buckling Analysis	-100					

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$$

Nx is now known, ε_x is unknown

FBD Math – Workspace Loads

Specifed Load

	Nx, εx	Ny, εy	Nxy, γxy	Mx, κx	My, κy	Mxy, κxy
Applied Unit Value	Load	Constrained	Constrained	Constrained	Constrained	Constrained
For Strength Analysis	-100					
For Buckling Analysis	-100					

$$\begin{Bmatrix} \varepsilon_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A' & B' \\ B' & D' \end{bmatrix} \begin{Bmatrix} N_x \\ \varepsilon_y \\ \varepsilon_{xy} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$$

Switch N_x and ε_x - Rearrange ABD

FBD Math – Workspace Loads

Specifed Load

	Nx, εx	Ny, εy	Nxy, γxy	Mx, κx	My, κy	Mxy, κxy
Applied Unit Value	Load	Constrained	Constrained	Constrained	Constrained	Constrained
For Strength Analysis	-100					
For Buckling Analysis	-100					

$$\begin{Bmatrix} \varepsilon_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A' & B' \\ B' & D' \end{bmatrix} \begin{Bmatrix} -100 \\ 0.0 \\ 0.0 \\ 0.0 \\ 0.0 \\ 0.0 \end{Bmatrix}$$

FBD Math – Workspace Loads

Virtual Loads

	Nx, εx	Ny, εy	Nxy, γxy	Mx, κx	My, κy	Mxy, κxy
Applied Unit Value	Load	Constrained	Constrained	Constrained	Constrained	Constrained
For Strength Analysis	-100					
For Buckling Analysis	-100					

$$\begin{Bmatrix} \varepsilon_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A' & B' \\ B' & D' \end{bmatrix} \begin{Bmatrix} N_x \\ \varepsilon_y \\ \varepsilon_{xy} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$$

Controlling Analysis Load: STRENGTH	Nx, εx	Ny, εy	Nxy, γxy	Mx, κx	My, κy	Mxy, κxy
Virtual Loads	0	-31	0	0	0	0
Design-to Loads	-100	-31	0	0	0	0
Design-to Deformation	-4.351252E-05	0	0	0	0	0

FBD Math – Workspace Loads

Virtual Loads

	Nx, εx	Ny, εy	Nxy, γxy	Mx, κx	My, κy	Mxy, κxy
Applied Unit Value	Load	Constrained	Constrained	Constrained	Constrained	Constrained
For Strength Analysis	-100					
For Buckling Analysis	-100					

$$\begin{Bmatrix} \varepsilon_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A' & B' \\ B' & D' \end{bmatrix} \begin{Bmatrix} N_x \\ \varepsilon_y \\ \varepsilon_{xy} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$$

Controlling Analysis Load: STRENGTH	Nx, εx	Ny, εy	Nxy, γxy	Mx, κx	My, κy	Mxy, κxy
Virtual Loads	0	-31	0	0	0	0
Design-to Loads	-100	-31	0	0	0	0
Design-to Deformation	-4.351252E-05	0	0	0	0	0

FBD Math – Workspace Loads

Virtual Loads

	Nx, εx	Ny, εy	Nxy, γxy	Mx, κx	My, κy	Mxy, κxy
Applied Unit Value	Load	Constrained	Constrained	Constrained	Constrained	Constrained
For Strength Analysis	-100					
For Buckling Analysis	-100					

$$\begin{Bmatrix} \varepsilon_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A' & B' \\ B' & D' \end{bmatrix} \begin{Bmatrix} N_x \\ \varepsilon_y \\ \varepsilon_{xy} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$$

Controlling Analysis Load: STRENGTH	Nx, εx	Ny, εy	Nxy, γxy	Mx, κx	My, κy	Mxy, κxy
Virtual Loads	0	-31	0	0	0	0
Design-to Loads	-100	-31	0	0	0	0
Design-to Deformation	-4.351252E-05	0	0	0	0	0

FBD Math – Workspace Loads

Virtual Loads

$$N_y = A'_{21} N_x + A'_{22} \varepsilon_y + \ldots$$

$$\begin{Bmatrix} \varepsilon_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A' & B' \\ B' & D' \end{bmatrix} \begin{Bmatrix} N_x \\ \varepsilon_y \\ \varepsilon_{xy} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$$

Controlling Analysis Load: STRENGTH	Nx, εx	Ny, εy	Nxy, γxy	Mx, κx	My, κy	Mxy, κxy
Virtual Loads	0	-31	0	0	0	0
Design-to Loads	-100	-31	0	0	0	0
Design-to Deformation	-4.351252E-05	0	0	0	0	0

Free Boundary Conditions

	N_x, ε_x	N_y, ε_y	N_{xy}, γ_{xy}	M_x, κ_x	M_y, κ_y	M_{xy}, κ_{xy}
Applied Unit Value	Load	Free	Constrained	Constrained	Constrained	Constrained
For Strength Analysis	-100					
For Buckling Analysis	-100					

$$\begin{Bmatrix}\varepsilon_x\\ N_y\\ N_{xy}\\ M_x\\ M_y\\ M_{xy}\end{Bmatrix} = \begin{bmatrix}A' & B'\\ B' & D'\end{bmatrix}\begin{Bmatrix}N_x\\ \varepsilon_y\\ \varepsilon_{xy}\\ \kappa_x\\ \kappa_y\\ \kappa_{xy}\end{Bmatrix}$$

Controlling Analysis Load: STRENGTH	N_x, ε_x	N_y, ε_y	N_{xy}, γ_{xy}	M_x, κ_x	M_y, κ_y	M_{xy}, κ_{xy}
Virtual Loads						
Design-to Loads	-100	0	0	0	0	0
Design-to Deformation	-5.212941E-05	1.616012E-05	0	0	0	0

Free Boundary Conditions

	Nx, εx	Ny, εy	Nxy, γxy	Mx, κx	My, κy	Mxy, κxy
Applied Unit Value	Load	Free	Constrained	Constrained	Constrained	Constrained
For Strength Analysis	-100					
For Buckling Analysis	-100					

$$\begin{Bmatrix} \varepsilon_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A' & B' \\ B' & D' \end{bmatrix} \begin{Bmatrix} N_x \\ \varepsilon_y \\ \varepsilon_{xy} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$$

Controlling Analysis Load: STRENGTH	Nx, εx	Ny, εy	Nxy, γxy	Mx, κx	My, κy	Mxy, κxy
Virtual Loads						
Design-to Loads	-100	0	0	0	0	0
Design-to Deformation	-5.212941E-05	1.616012E-05	0	0	0	0

Free Boundary Conditions

	Nx, εx	Ny, εy	Nxy, γxy	Mx, κx	My, κy	Mxy, κxy
Applied Unit Value	Load	Free	Constrained	Constrained	Constrained	Constrained
For Strength Analysis	-100					
For Buckling Analysis	-100					

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A'' & B'' \\ B'' & D'' \end{bmatrix} \begin{Bmatrix} N_x \\ N_y = 0 \\ \varepsilon_{xy} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$$

Controlling Analysis Load: STRENGTH	Nx, εx	Ny, εy	Nxy, γxy	Mx, κx	My, κy	Mxy, κxy
Virtual Loads						
Design-to Loads	-100	0	0	0	0	0
Design-to Deformation	-5.212941E-05	1.616012E-05	0	0	0	0

Free Boundary Conditions

	Nx, εx	Ny, εy	Nxy, γxy	Mx, κx	My, κy	Mxy, κxy
Applied Unit Value	Load	Free	Constrained	Constrained	Constrained	Constrained
For Strength Analysis	-100					
For Buckling Analysis	-100					

$$
\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix}
=
\begin{bmatrix} A'' & B'' \\ B'' & D'' \end{bmatrix}
\begin{Bmatrix} N_x \\ N_y = 0 \\ \varepsilon_{xy} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}
$$

Controlling Analysis Load: STRENGTH	Nx, εx	Ny, εy	Nxy, γxy	Mx, κx	My, κy	Mxy, κxy
Virtual Loads						
Design-to Loads	-100	0	0	0	0	0
Design-to Deformation	-5.212941E-05	1.616012E-05	0	0	0	0

Free Boundary Conditions

	Nx, εx	Ny, εy	Nxy, γxy	Mx, κx	My, κy	Mxy, κxy
Applied Unit Value	Load	Free	Constrained	Constrained	Constrained	Constrained
For Strength Analysis	-100					
For Buckling Analysis	-100					

$$
\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix}
=
\begin{bmatrix} A'' & B'' \\ B'' & D'' \end{bmatrix}
\begin{Bmatrix} N_x \\ N_y = 0 \\ \varepsilon_{xy} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}
$$

Note: double prime 6x6 matrix

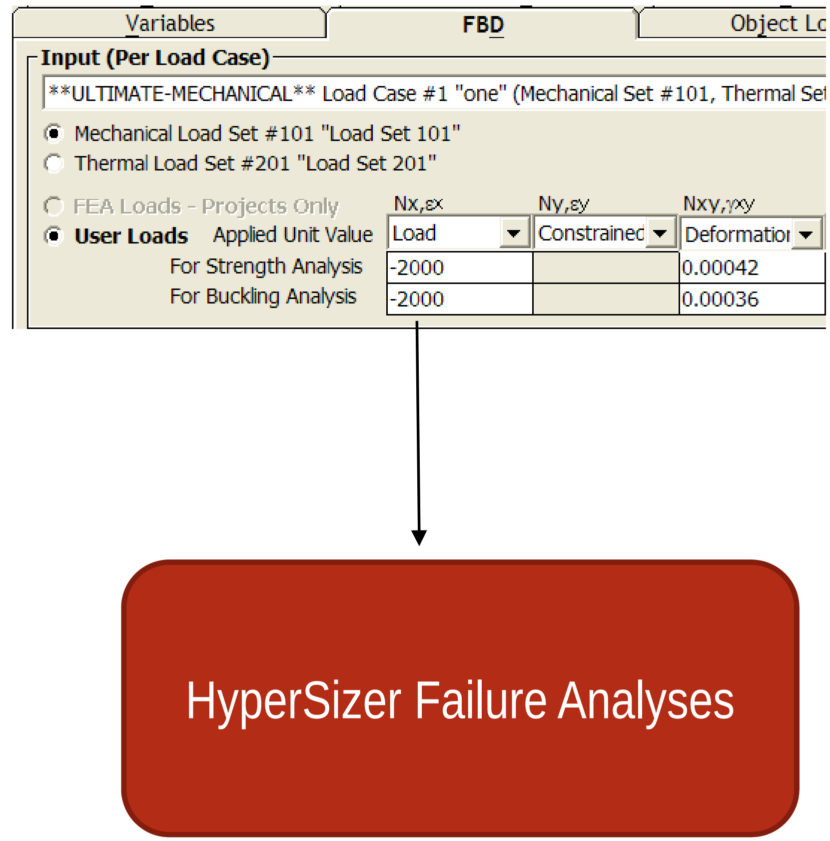
Controlling Analysis Load: STRENGTH	Nx, εx	Ny, εy	Nxy, γxy	Mx, κx	My, κy	Mxy, κxy
Virtual Loads						
Design-to Loads	-100	0	0	0	0	0
Design-to Deformation	-5.212941E-05	1.616012E-05	0	0	0	0

Analyses are Independent of Loads Source

HyperSizer Failure Analyses

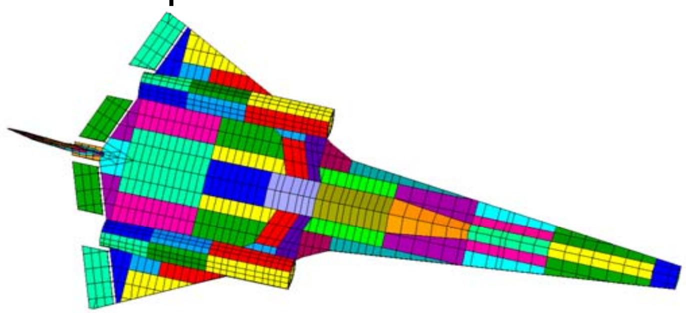
Analyses are Independent of Loads Source

User Input by **hand,** (typed-in loads).
Very convenient interactive tool

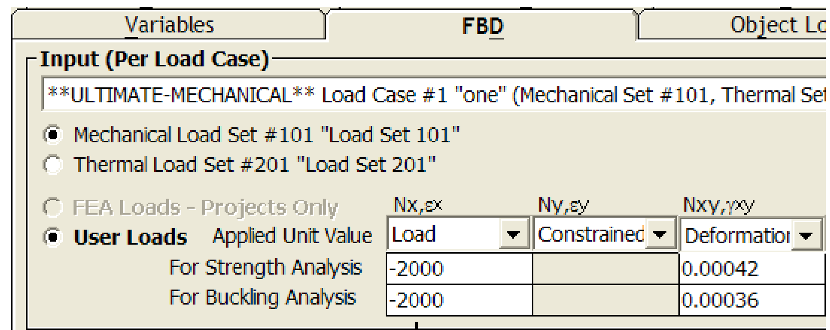


Analyses are Independent of Loads Source

FEA Computed Loads



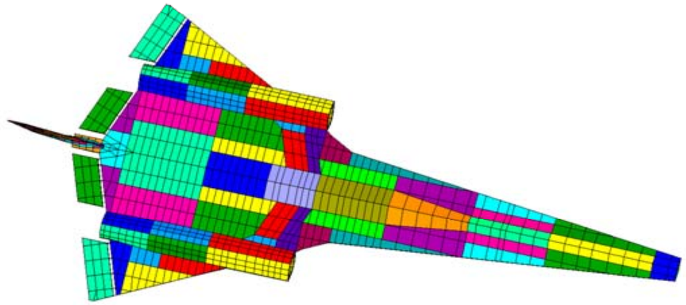
User Input by **hand,** (typed-in loads).
Very convenient interactive tool



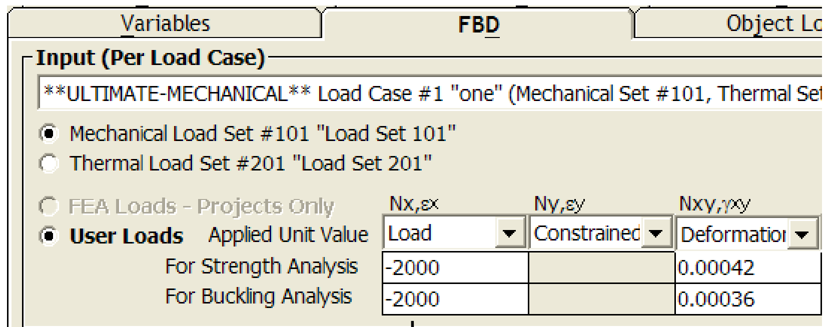
HyperSizer Failure Analyses

Analyses are Independent of Loads Source

FEA Computed Loads



User Input by **hand,** (typed-in loads).
Very convenient interactive tool



Other Sources Using HyperSizer's Object Model Interface:

- Loads from spreadsheets
- Loads from a larger company software design system

HyperSizer Failure Analyses

Appendix

Appendix I: ABD of Isotropic Plate

- Reduced stiffness matrix Q
  - Plane stress constitutive equation
  - In-plane properties E, v, & G

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} \dfrac{E}{1-\nu^2} & \dfrac{\nu E}{1-\nu^2} & 0 \\ \dfrac{\nu E}{1-\nu^2} & \dfrac{E}{1-\nu^2} & 0 \\ 0 & 0 & G_{12} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} = \mathbf{Q}\vec{\varepsilon}$$

Appendix I: ABD of Isotropic Plate

- Integrate Q over the single layer
  - h_0 = -t/2, h_1 = t/2

$$A_{ij} = \sum_{k=1}^{n} Q_{ij}\left(\theta_k\right)\left(h_k - h_{k-1}\right) = \boxed{Qt}$$

$$B_{ij} = \sum_{k=1}^{n} Q_{ij}\left(\theta_k\right)\left(h_k^2 - h_{k-1}^2\right)/2 = Q\left(\frac{t^2}{4} - \frac{t^2}{4}\right)/2 = \boxed{0}$$

$$D_{ij} = \sum_{k=1}^{n} Q_{ij}\left(\theta_k\right)\left(h_k^3 - h_{k-1}^3\right)/3 = Q\left(\frac{t^3}{8} - -\frac{t^3}{8}\right)/3 = \boxed{Q\frac{t^3}{12}}$$

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Appendix I: ABD of Isotropic Plate

- Final ABD

$$\begin{bmatrix} N_x \\ N_y \\ N_s \\ M_x \\ M_y \\ M_s \end{bmatrix} = \begin{bmatrix} \frac{Et}{1-\nu^2} & \frac{\nu Et}{1-\nu^2} & 0 & 0 & 0 & 0 \\ \frac{\nu Et}{1-\nu^2} & \frac{Et}{1-\nu^2} & 0 & 0 & 0 & 0 \\ 0 & 0 & G_{12}t & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{Et^3}{12\left(1-\nu^2\right)} & \frac{\nu Et^3}{12\left(1-\nu^2\right)} & 0 \\ 0 & 0 & 0 & \frac{\nu Et^3}{12\left(1-\nu^2\right)} & \frac{Et^3}{12\left(1-\nu^2\right)} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{G_{12}t^3}{12} \end{bmatrix} \begin{bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \\ \kappa_x \\ \kappa_y \\ \kappa_z \end{bmatrix}$$

Appendix II: Iso Effective Elastic Constants

- Goal is reduce ABD relation of isotropic plate to the forms:
  - N_x load, N_y free, N_{xy} free, M free

$$\varepsilon_x = \frac{N_x}{tE_x^{eff}}$$

$$\varepsilon_y = -\nu_{xy}^{eff}\,\varepsilon_x$$

Appendix II: Iso Effective Elastic Constants

- Start with isotropic ABD

$$\begin{bmatrix} N_x \\ N_y \\ N_s \\ M_x \\ M_y \\ M_s \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 & 0 & 0 & 0 \\ A_{12} & A_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & A_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & D_{11} & D_{12} & 0 \\ 0 & 0 & 0 & D_{12} & D_{22} & 0 \\ 0 & 0 & 0 & 0 & 0 & D_{33} \end{bmatrix} \begin{bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \\ \kappa_x \\ \kappa_y \\ \kappa_z \end{bmatrix}$$

Appendix II: Iso Effective Elastic Constants

- Invert

$$\begin{bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \\ \kappa_x \\ \kappa_y \\ \kappa_z \end{bmatrix} = \begin{bmatrix} \frac{A_{22}}{A_{11}A_{22}-A_{12}^2} & \frac{-A_{12}}{A_{11}A_{22}-A_{12}^2} & 0 & 0 & 0 & 0 \\ \frac{-A_{12}}{A_{11}A_{22}-A_{12}^2} & \frac{A_{11}}{A_{11}A_{22}-A_{12}^2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{A_{33}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{D_{22}}{-D_{12}^2+D_{11}D_{22}} & \frac{-D_{12}}{-D_{12}^2+D_{11}D_{22}} & 0 \\ 0 & 0 & 0 & \frac{-D_{12}}{-D_{12}^2+D_{11}D_{22}} & \frac{D_{11}}{-D_{12}^2+D_{11}D_{22}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{D_{33}} \end{bmatrix} \begin{bmatrix} N_x \\ N_y \\ N_s \\ M_x \\ M_y \\ M_s \end{bmatrix}$$

Appendix II: Iso Effective Elastic Constants

- N_x load, N_y free, N_{xy} free, M free

$$\varepsilon_x^o = \frac{A_{22}}{A_{11}A_{22} - A_{12}^2} N_x$$

$$= \frac{N_x}{E_x^{eff}}$$

$$\varepsilon_y^o = \frac{-A_{12}}{A_{11}A_{22} - A_{12}^2} N_x$$

$$= \frac{-A_{12}}{A_{11}A_{22} - A_{12}^2} \frac{A_{11}A_{22} - A_{12}^2}{A_{22}} \varepsilon_x^o$$

$$= \frac{-A_{12}}{A_{22}} \varepsilon_x^o$$

$$= -\nu_{xy}^{eff} \varepsilon_x^o$$

Appendix III: ABD^{-1} of Isotropic Plate

- Inverted ABD – see Appendix II

$$
\begin{bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \\ \kappa_x \\ \kappa_y \\ \kappa_z \end{bmatrix}
=
\begin{bmatrix}
\frac{A_{22}}{A_{11}A_{22}-A_{12}^2} & \frac{-A_{12}}{A_{11}A_{22}-A_{12}^2} & 0 & 0 & 0 & 0 \\
\frac{-A_{12}}{A_{11}A_{22}-A_{12}^2} & \frac{A_{11}}{A_{11}A_{22}-A_{12}^2} & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{A_{33}} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{D_{22}}{-D_{12}^2+D_{11}D_{22}} & \frac{-D_{12}}{-D_{12}^2+D_{11}D_{22}} & 0 \\
0 & 0 & 0 & \frac{-D_{12}}{-D_{12}^2+D_{11}D_{22}} & \frac{D_{11}}{-D_{12}^2+D_{11}D_{22}} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{D_{33}}
\end{bmatrix}
\begin{bmatrix} N_x \\ N_y \\ N_s \\ M_x \\ M_y \\ M_s \end{bmatrix}
$$

Appendix III: ABD^{-1} of Isotropic Plate

- Simplify inverse matrix terms

$$A_{11}^{-1} = \frac{A_{22}}{A_{11}A_{22} - A_{12}^2} = \frac{Et}{\frac{(Et)^2 - (Et\nu)^2}{1-\nu^2}} = \frac{1-\nu^2}{Et\left(1-\nu^2\right)} = \frac{1}{Et}$$

$$A_{22}^{-1} = \frac{1}{Et}$$

$$A_{12}^{-1} = \frac{-\nu}{Et}$$

$$A_{33}^{-1} = \frac{1}{Gt}$$

$$\begin{bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \\ \kappa_x \\ \kappa_y \\ \kappa_z \end{bmatrix} = \begin{bmatrix} A_{11}^{-1} & A_{12}^{-1} & 0 & 0 & 0 & 0 \\ A_{12}^{-1} & A_{22}^{-1} & 0 & 0 & 0 & 0 \\ 0 & 0 & A_{33}^{-1} & 0 & 0 & 0 \\ 0 & 0 & 0 & D_{11}^{-1} & D_{12}^{-1} & 0 \\ 0 & 0 & 0 & D_{12}^{-1} & D_{22}^{-1} & 0 \\ 0 & 0 & 0 & 0 & 0 & D_{33}^{-1} \end{bmatrix} \begin{bmatrix} N_x \\ N_y \\ N_s \\ M_x \\ M_y \\ M_s \end{bmatrix}$$

Extra

Isotropic Plate Stiffness

- Compliance

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} \frac{1}{E} & \frac{-\nu}{E} & 0 \\ \frac{-\nu}{E} & \frac{1}{E} & 0 \\ 0 & 0 & \frac{1}{G_{12}} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix}$$

Isotropic Plate Stiffness

- Compliance
- Stiffness

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} \frac{1}{E} & \frac{-\nu}{E} & 0 \\ \frac{-\nu}{E} & \frac{1}{E} & 0 \\ 0 & 0 & \frac{1}{G_{12}} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} \quad \Longleftrightarrow \quad \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} \frac{E}{1-\nu^2} & \frac{\nu E}{1-\nu^2} & 0 \\ \frac{\nu E}{1-\nu^2} & \frac{E}{1-\nu^2} & 0 \\ 0 & 0 & G_{12} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix}$$

HyperSizer®

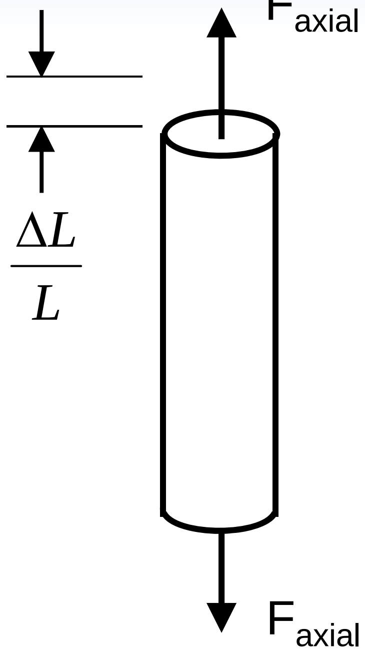
Isotropic Plate Stiffness

- Compliance
- Stiffness

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} \frac{1}{E} & \frac{-\nu}{E} & 0 \\ \frac{-\nu}{E} & \frac{1}{E} & 0 \\ 0 & 0 & \frac{1}{G_{12}} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} \Longleftrightarrow \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} \frac{E}{1-\nu^2} & \frac{\nu E}{1-\nu^2} & 0 \\ \frac{\nu E}{1-\nu^2} & \frac{E}{1-\nu^2} & 0 \\ 0 & 0 & G_{12} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix}$$

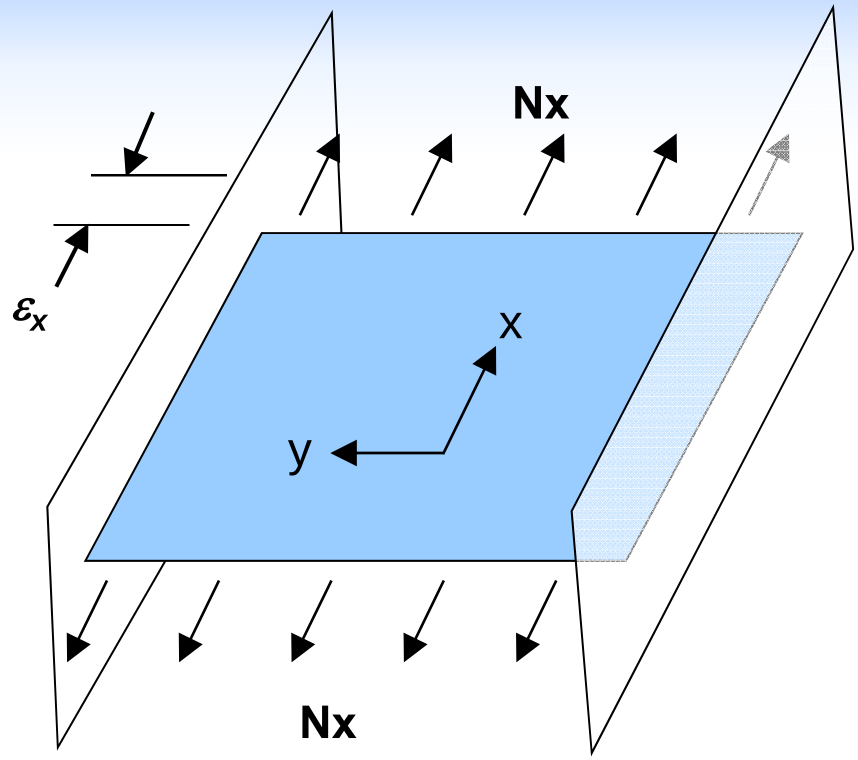
Poisson term for plates $\dfrac{1}{1-\nu^2}$

Effective Modulus ($\varepsilon_y = 0$)



$$\varepsilon = \frac{\Delta L}{L}$$

$$\sigma = E\varepsilon$$



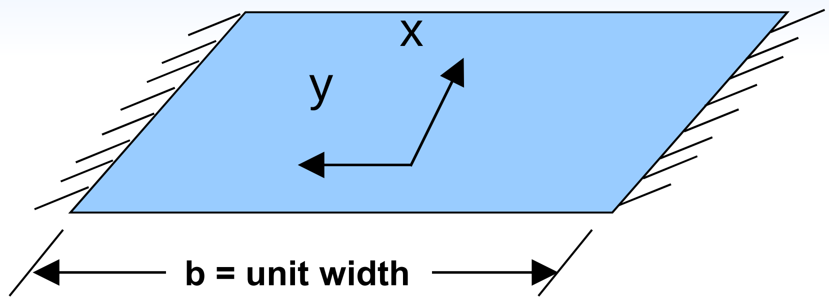
$$\sigma_x = \left(\frac{E}{1-\nu^2}\right)\varepsilon_x + \nu\cancel{\left(\frac{E}{1-\nu^2}\right)\varepsilon_y}$$

$$\sigma_x = \left(\frac{E}{1-\nu^2}\right)\varepsilon_x = E^*\varepsilon_x$$

Convert Stress → Line Load

$$N_x = \sigma_x t$$

$$\sigma_x = E^* \varepsilon_x$$



Area

$$A = bt$$

Moment of Inertia

$$I = \frac{bt^3}{12}$$

$$M_x' = M_x t$$

$$M_x = E^* I \kappa$$

Membrane

Unit Force

$$N_x^* = \left(\frac{E}{1-\nu^2}\right)\varepsilon t$$

$$= \frac{Et}{1-\nu^2}\varepsilon_x$$

$$= A_{11}\varepsilon_x$$

Stiffness

$$\boxed{A_{11} = \frac{Et}{1-\nu^2}}$$

Bending

Unit Moment

$$M_x' = \left(\frac{E}{1-\nu^2}\right) I \kappa t$$

$$= \frac{Et^3}{12\left(1-\nu^2\right)}\kappa$$

$$= D_{11}\kappa$$

Bending Stiffness

$$\boxed{D_{11} = \frac{Et^3}{12\left(1-\nu^2\right)}}$$

HyperSizer®

Membrane Coupling Relationships

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \begin{bmatrix} \frac{Et}{1-\nu^2} & \frac{\nu Et}{1-\nu^2} & 0 \\ \frac{\nu Et}{1-\nu^2} & \frac{Et}{1-\nu^2} & 0 \\ 0 & 0 & Gt \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}$$

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \frac{1}{Et} & -\frac{\nu}{Et} & 0 \\ -\frac{\nu}{Et} & \frac{1}{Et} & 0 \\ 0 & 0 & \frac{1}{Gt} \end{bmatrix} \begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix}$$